

**Answer to Essential Question 14.6:** 600 K. An isotherm is a line connecting all the points satisfying the equation  $PV = nRT =$  a particular constant that depends on  $n$  and  $T$ . Because we're talking about the same line on both  $P$ - $V$  diagrams, we have  $PV = n_A RT_A = n_B RT_B$ . Solving for the temperature in cylinder  $B$  gives:

$$T_B = \frac{n_A RT_A}{n_B R} = \frac{n_A}{n_B} T_A = \frac{2n_B}{n_B} T_A = 2T_A = 2(300\text{K}) = 600\text{K}.$$

In this sense, then, the  $P$ - $V$  diagrams for different ideal gas systems are unique, because the temperature of a particular isotherm depends on the number of moles of gas in the system.

## Chapter Summary

### *Essential Idea regarding looking at thermodynamic systems on a microscopic level*

We can apply basic principles of physics to a system of gas molecules, at the microscopic level, and get important insights into macroscopic properties such as temperature. Temperature is a measure of the average kinetic energy of the atoms or molecules of the gas.

### *The Ideal Gas Law*

The ideal gas law can be written in two equivalent forms.

In terms of  $n$ , the number of moles of gas,  $PV = nRT$ , (Equation 14.1)

where  $R = 8.31 \text{ J/(mol K)}$  is the universal gas constant.

In terms of  $N$ , the number of molecules,  $PV = NkT$ , (Equation 14.2) where

$k = 1.38 \times 10^{-23} \text{ J/K}$  is Boltzmann's constant.

### *What Temperature Means*

$$K_{av} = \frac{3}{2} kT. \quad (\text{Equation 14.14: Average kinetic energy is directly related to temperature})$$

As Equation 14.14 shows, temperature is a direct measure of the average kinetic energy of the atoms or molecules in the ideal gas.

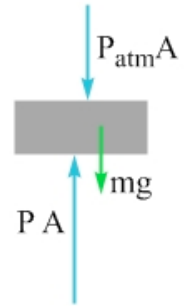
### *The Maxwell-Boltzmann Distribution*

The Maxwell-Boltzmann distribution is the distribution of molecular speeds in a container of ideal gas, which depends on the molar mass  $M$  of the molecules and on the absolute temperature,  $T$ . The distribution is characterized by three speeds. In decreasing order, these are the root-mean-square speed; the average speed; and the most-probable speed. These are given by equations 14.16 – 14.18:

$$v_{rms} = \sqrt{\frac{3RT}{M}}; \quad v_{av} = \sqrt{\frac{8RT}{\pi M}}; \quad v_{prob} = \sqrt{\frac{2RT}{M}}.$$

***A Cylinder Sealed by a Piston that can Move Without Friction***

A common example of an ideal gas system is ideal gas sealed inside a cylinder by means of a piston that is free to move without friction. When the piston is at its equilibrium position the pressure of the gas is generally determined by balancing the forces on the piston’s free-body diagram, rather than from the volume or temperature of the gas. The diagram at right illustrates this idea for a cylinder sealed at the top by a piston of area  $A$ . The combined forces directed down, the force and gravity and the force associated with atmospheric pressure acting on the top of the piston, must be balanced by the upward force associated with the gas pressure acting on the bottom of the piston.



***The Equipartition Theorem***

The equipartition theorem is the idea that each contribution to the internal energy (energy associated with the motion of the molecules) of an ideal gas contributes equally. Each contribution is known as a degree of freedom.

The energy from each degree of freedom =  $\frac{1}{2} Nkt = \frac{1}{2} nRt$ . (Equation 14.20)

A monatomic ideal gas can experience translational motion in three dimensions. With three degrees of freedom the internal energy is given by:

$E_{int} = NK_{av} = \frac{3}{2} NkT = \frac{3}{2} nRT$ . (Eq. 14.19: **Internal energy of a monatomic ideal gas**)

At intermediate temperatures molecules in a diatomic ideal gas have two additional degrees of freedom, associated with rotation about two axes.

$E_{int} = \frac{5}{2} NkT = \frac{5}{2} nRT$ . (Eq. 14.21: **Internal energy of a diatomic ideal gas**)

Molecules in a polyatomic ideal gas can rotate about three axes.

$E_{int} = \frac{6}{2} NkT = 3NkT = 3nRT$ . (Eq. 14.22: **Internal energy of a polyatomic ideal gas**)

***The P-V Diagram***

A graph of pressure versus volume (a P-V diagram) can be very helpful in understanding an ideal gas system. We will exploit these even more in the next chapter. The ideal gas law tells us that the product of pressure and volume (which has units of energy) is proportional to the temperature of a system. Lines of constant temperature are known as isotherms. The diagram at right shows two isotherms. The isotherm that is farther from the origin has twice the absolute temperature as the isotherm closer to the origin.

