Answer to Essential Question 13.2: The key here is that aluminum has a larger expansion coefficient than iron (see Table 13.1), so the aluminum expands or contracts more than does the iron ring for the same change in temperature. Increasing the temperature would make the mismatch worse but, if we decrease the temperature enough, both objects will contract, with the aluminum contracting more so the iron ring will slide onto the cylinder.

13-3 Specific Heat

Heat is energy transferred between objects that are at different temperatures, transferred from the higher-temperature object to the lower-temperature object. Although this is a form of energy we have not discussed in previous chapters, we will deal with it in much the same way we dealt with other forms of energy. Conservation of energy, for instance, will be a very useful tool, as it was previously.

The type of question we'll deal with next is the following: what is the equilibrium temperature when a 500-gram lead ball at 100°C is added to 400 g of water that is at room temperature, 20°C? To answer this kind of question we will assume that no energy is transferred into or out of the system, and we will also use a simple model that says that when energy in the form of heat is added to or removed from a substance, the substance's change in temperature is proportional to the amount of heat added or removed. The equation that goes with this statement is the following, where the symbol Q is used to represent heat, with units of energy:

$$Q = m c \Delta T \,, \qquad \text{(Eq. 13.8: \textbf{The heat required to change an object's temperature})}$$

where m is the mass of the substance, c is known as the specific heat capacity (a constant that depends on the material), and ΔT is the temperature change. Note that, if heat is added to a substance, Q and ΔT are both positive; if heat is removed, Q and ΔT are both negative. Table 13.2 shows values of specific heat capacity for various materials. Equation 13.8 is generally valid as long as the substance does not change phase (such as from solid to liquid). We will learn how to deal with phase changes in the next section.

Material	**c, specific heat capacity (J / kg °C)**
Aluminum	900
Brass	377
Copper	385
Gold	128
Iron	449
Lead	129
Water (gas)	1850
Water (liquid)	4186
Water (solid)	2060

Table 13.2: Specific heat capacities for several solid metals, and for three common forms of water. The value for gaseous water is valid at standard atmospheric pressure.

Specific heat capacities are sometimes given in calories / (g °C). One calorie is the amount of heat needed to change the temperature of 1 g of water by 1°C. Thus 1000 cal is needed to change the temperature of 1 kg (1000 g) of water by 1°C. Looking at the specific heat capacity of liquid water in Table 13.2, we can see that 4186 J is equivalent to 1000 cal, giving us a conversion factor of 4.186 J/cal.

EXPLORATION 13.3 – The large heat capacity of water

A 500-gram lead ball that is initially at 100°C is added to an unknown mass of water that is initially at room temperature, 20°C, in a Styrofoam cup. After allowing the system to come to thermal equilibrium (i.e., waiting until the lead ball and the water have the same temperature), the final temperature of the system is measured to be 60°C, exactly halfway between the initial temperatures of the lead ball and the water. Based on this, determine the mass of water in the system.

Step 1 – ***Using Table 13.2, use a qualitative argument to predict how the mass of water compares to the mass of the lead ball.*** Both the lead and the water experience a temperature change of the same magnitude, with the temperature of the lead falling 40°C and the water temperature rising 40°C. If their heat capacities were equal, therefore, their masses would also be equal. From Table 13.2, however, we see that their heat capacities differ by a large factor. The heat capacity of lead (chemical symbol Pb) is $c_{Pb} = 129 \text{ J/kg °C}$ while the heat capacity of water (H_2O), is much larger, at $c_{H_2O} = 4186 \text{ J/kg °C}$. Refer to equation 13.8, $Q = m c \Delta T$. The heat Q has the same magnitude for both, because it is the energy transferred from the lead to the water. Therefore, the larger the specific heat capacity, the smaller the mass. In this case, then, the mass of water must be considerably smaller than the mass of the lead ball.

Step 2 – ***Use equation 13.8, and energy conservation, to find the mass of water in the system.*** In the static equilibrium problems we analyzed in Chapter 10 we used the equations $\sum \vec{F} = 0$ and $\sum \vec{\tau} = 0$. We can apply a similar equation in this thermal equilibrium situation:

$$\sum Q = 0 . \qquad \text{(Equation 13.9: \textbf{An equation for thermal equilibrium})}$$

This is really a simple statement of conservation of energy. In the case of the lead ball and the water, it means that all the heat transferred out of the lead ball (this Q is negative) is transferred to the water (that Q is positive). Combining equations 13.8 and 13.9 gives:

$$Q_{Pb} + Q_{H_2O} = 0 ;$$

$$m_{Pb}\, c_{Pb}\, \Delta T_{Pb} + m_{H_2O}\, c_{H_2O}\, \Delta T_{H_2O} = 0 .$$

Solving for the unknown mass of the water gives:

$$m_{H_2O} = \frac{-m_{Pb}\, c_{Pb}\, \Delta T_{Pb}}{c_{H_2O}\, \Delta T_{H_2O}} = \frac{-m_{Pb}\, c_{Pb} \left(T_{f,Pb} - T_{i,Pb}\right)}{c_{H_2O} \left(T_{f,H_2O} - T_{i,H_2O}\right)} ,$$

where the subscripts i and f on the temperatures correspond to the initial and final values, respectively. Plugging in the numerical values gives:

$$m_{H_2O} = \frac{-(500\text{ g})(129 \text{ J/kg °C})(60\text{°C} - 100\text{°C})}{(4186 \text{ J/kg °C})(60\text{°C} - 20\text{°C})} = 15.4 \text{ g} .$$

As we predicted, the mass of water is much smaller than the mass of the lead ball. This is because of the unusually large specific heat capacity of liquid water.

Key ideas regarding thermal equilibrium: In a thermal equilibrium situation we can use a simple energy conservation relationship, $\sum Q = 0$. We also observed that liquid water has a large specific heat capacity. **Related End-of-Chapter Exercises: 22 – 25.**

Note that, in Exploration 13.3, there is no need to convert the temperatures from Celsius to Kelvin, because the equation for Q involves ΔT and not T. In addition, there is no need to convert the given mass of the lead ball from grams to kilograms, even though the specific heat capacities are specified in kg. In the end, the units in the two specific heats cancel out, so the mass of the water comes out in the units the mass of the lead ball is given in.

Essential Question 13.3: Many people learn to do thermal equilibrium problems by setting the heat lost by one or more parts of a system equal to the heat gained by other parts. Compare and contrast that method to the $\sum Q = 0$ used in Exploration 13.3.