PROBLEM 1 – 10 points

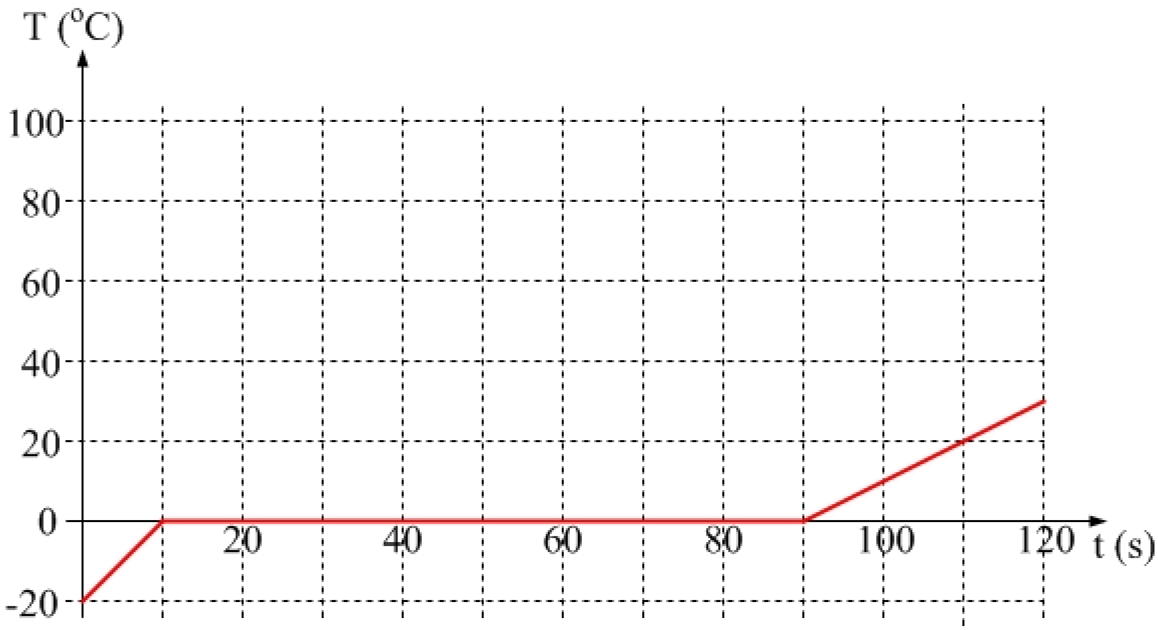
Heat is added to a sample of water at a constant rate over a two-minute period.

[7 points] (a) Using the axes provided below, draw a graph that represents the plot of the temperature of the sample vs. time over this two-minute period.

The sample, which is initially solid, starts at -20°C and it takes 10 seconds to reach 0°C.

You may find the following data helpful when drawing the graph:

Specific heats for water: c_{liquid} = 1.0 cal/g °C and $c_{ice} = c_{steam}$ = 0.5 cal/g °C
Latent heats for water: heat of fusion L_f = 80 cal/g and heat of vaporization L_v = 540 cal/g



In the first 10 seconds, this much heat is added to the sample: $Q = mc\Delta T = m \times 10$ cal/g **.**

The power is $P = Q/t = m \times 1$ cal/(g s) **. We can use this to figure out how long it takes to melt the ice:** $Pt = Q = mL_f \quad \Rightarrow \quad t = mL_f / P = 80$ s **. In the last 30 s, therefore, the temperature of the liquid water increases. The change in temperature over this time period is given by:**

$$Pt = Q = mc\Delta T \quad \Rightarrow \quad \Delta T = Pt/mc = 30/1 = 30°C$$

[3 points] (b) If heat is being added to the sample at a constant rate of 90 cal/s, what is the mass of the sample?

Let's use the equation we derived above, $P = Q/t = m \times 1$ cal/(g s) **.**
With *P* = 90 cal/s, we get *m* = 90 g.

PROBLEM 2 – 8 points

In preparing a cold drink with a final temperature of 0°C, Mary mixes 50 g of ice at –10 °C with some juice at +10 °C. Suppose the specific heat of the juice is 4000 J/(kg °C), the specific heat of ice is 2000 J/(kg °C) and the latent heats of fusion of water and the juice are the same, equal to 3×10^5 J/kg. Answer the following questions based on the information provided. You may assume that the cup Mary uses in making the drink insulates well so there is no heat loss during the mixing, i.e., ΣQ = 0.

[4 points] (a) What mass of juice should Mary add to the ice so that no ice is transformed into liquid water and no liquid juice is transformed into solid juice?

With no phase change, we get:

$$m_{ice}c_{solid}\Delta T + m_{juice}c_{liquid}\Delta T = 0$$

$$(50\text{ g})\times[2000\text{ J/(kg °C)}]\times(+10°\text{C}) + m_{juice}\times[4000\text{ J/(kg °C)}]\times(-10°\text{C}) = 0$$

This gives $m_{juice} = 25\text{ g}$

[4 points] (c) What mass of juice should Mary add to the ice so that all the ice is just melted?

In this case, the equation needs to account for the phase change:

$$m_{ice}c_{solid}\Delta T + m_{ice}L_f + m_{juice}c_{liquid}\Delta T = 0$$

$$(50\text{ g})\times[2000\text{ J/(kg °C)}]\times(+10°\text{C}) + (50\text{ g})\times(300000\text{ J/kg}) + m_{juice}\times[4000\text{ J/(kg °C)}]\times(-10°\text{C}) = 0$$

This gives $m_{juice} = \dfrac{(100+1500)\text{ g J/kg}}{4\text{ J/kg}} = 400\text{ g}$