Answer to Essential Question 12.5: We cannot tell. Any one of the three graphs can be used to determine that the angular frequency has changed, because they all involve ω, but none of the graphs can tell us whether we adjusted the spring constant or the mass.

12-6 Examples Involving Simple Harmonic Motion

EXAMPLE 12.6A – Energy graphs

Take a 0.500-kg block and attach it to a spring. We would like the block to undergo oscillations that have a period (the time for one complete oscillation) of 4.00 seconds.

(a) What should the spring constant be?

(b) We'll release the block from rest from a distance A from the equilibrium point so that the block has a speed of 4.00 m/s when it passes through equilibrium. Over two complete oscillations, plot the system's elastic potential energy, kinetic energy, and total mechanical energy as a function of time.

SOLUTION

(a) Let's first apply Equation 12.7, $\omega = 2\pi / T$, to find the angular frequency. This gives:

$$\omega = \frac{2\pi \text{ rad}}{4.00 \text{ s}} = \frac{\pi}{2.00} \text{ rad/s}.$$

Using Equation 12.7, $\omega = \sqrt{\frac{k}{m}}$, we get:

$$k = \omega^2 m = \frac{\pi^2 (0.500)}{4.00} \text{rad}^2 \text{ kg/s}^2 = 1.23 \text{ N/m},$$

where we treated the factor of radians as being dimensionless.

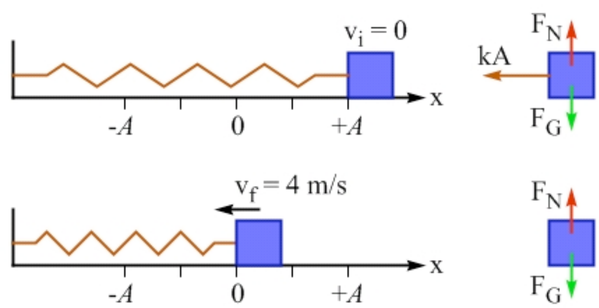

Figure 12.14: A diagram of the block and spring, showing the initial situation and the situation as the block passes through equilibrium.

(b) A diagram of the situation is shown in Figure 12.14. Let's solve for the maximum kinetic energy, which equals the mechanical energy:

$$K_{max} = \frac{1}{2} m v_{max}^2 = \frac{1}{2}(0.500 \text{ kg})(4.00 \text{ m/s})^2 = 4.00 \text{ J}.$$

The maximum potential energy is also 4.00 J, because the energy oscillates between potential and kinetic, and the total mechanical energy is conserved.

Using Equation 12.4, $\vec{v} = -v_{max} \sin(\omega t)$, we can write the kinetic energy as a function of time as $K = \frac{1}{2} m v^2 = \frac{1}{2} m v_{max}^2 \sin^2(\omega t) = (4.00 \text{ J}) \sin^2\left(\frac{\pi}{2.00 \text{ s}} t\right)$.

Because the block takes 4.00 s to complete one oscillation, at $t = 0$ and $t = 4.00$ s it is instantaneously at rest at the starting point. At $t = 2.00$ s (halfway through the cycle) the block is instantaneously at rest on the far side of equilibrium. At each of these times the kinetic energy is 0 and the elastic potential energy is 4.00 J. Conversely, at $t = 1.00$ s and 3.00 s it passes through equilibrium, where the elastic potential energy is zero and the kinetic energy is its maximum value of 4.00 J. Graphs of the various energies as a function of time are shown in Figure 12.15. Note that, at all times, the sum of the kinetic and potential energies is 4.00 J.

Related End-of-Chapter Exercises: 27, 40.

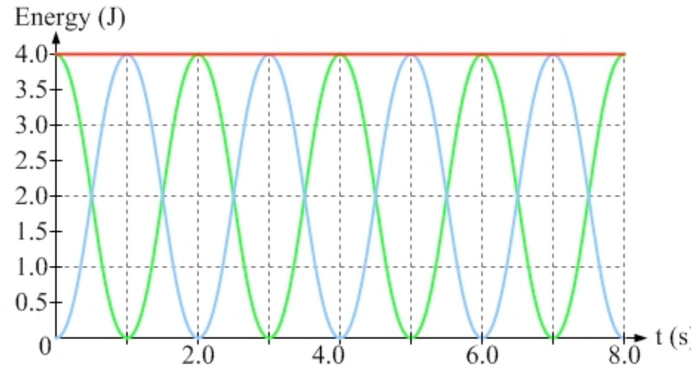

Figure 12.15: Graphs of the block's kinetic energy (zero at t = 0 s), elastic potential energy (zero at t = 1.0 s), and total mechanical energy (constant), as a function of time over 8.00 s, the time for two complete oscillations. The kinetic and potential energies go through two cycles for every one complete oscillation of the block. Compare this graph to Figure 12.5, which shows the energies as a function of position.

EXAMPLE 12.6B – An eighth of the motion

Attach an object to an ideal spring and set it oscillating. The object is released from rest from a distance A from equilibrium. The object travels a distance A to the equilibrium position, ¼ of the entire distance for one complete oscillation, in ¼ of the period. How long does it take the object to travel from A away from equilibrium to A/2 from equilibrium, 1/8 of the entire distance covered in one complete oscillation?

SOLUTION

A diagram of the situation is shown in Figure 12.16. Position 1 is where the block is released from rest. Position 2 is halfway between the release point and the equilibrium position, which is position 3.

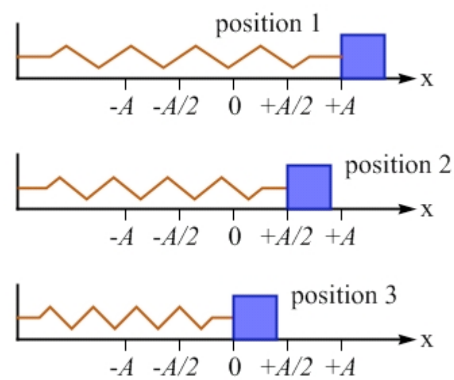

Figure 12.16: A diagram of the position of the block as it moves from its release point (position 1) to the equilibrium position (position 3). Position 2 is exactly halfway between the release point and equilibrium.

Because the block's speed increases as the block approaches equilibrium, the block's average speed as it moves from position 1 to position 2 is less than its average speed as it moves from position 2 to position 3. Thus, because of this low average speed, the time it takes to move from position 1 to position 2 is larger than T/8, 1/8 of the total time.

(b) Using Equation 12.3, $\vec{x} = A\cos(\omega t)$, let's find the time it takes. At position 2, the block's position is +A/2. Using this in Equation 12.3 gives:

$+\frac{A}{2} = A\cos(\omega t)$. Dividing by A gives: $+\frac{1}{2} = \cos(\omega t)$.

We can use the relation $\omega = 2\pi / T$ to re-write the equation: $+\frac{1}{2} = \cos\left(\frac{2\pi}{T}t\right)$.

The logical next step is to take the inverse cosine of both sides. Here it is critical to remember that ωt has units of radians. ***Any time we use the three equations involving time (Equations 12.3 – 12.5), we need to work in radians.*** Thus, when we determine $\cos^{-1}(+1/2)$ we will write the result as π/3 radians instead of 60˚.

Taking the inverse cosine of both sides, then, gives: $\frac{\pi}{3} = \frac{2\pi}{T}t$.

This gives us a time of t = T/6. As we concluded above, the time is larger than T/8.

Related End-of-Chapter Exercises: 34, 50.

Essential Question 12.6: Return to the situation described in Example 12.6A, but now increase the angular frequency by a factor of 2. Let's say we achieve the change in angular frequency by changing either the spring constant or the mass, but not both. Can we tell which one was changed, if the graphs of energy as a function of time still reach a maximum of 4.0 J when the block is released from rest from a distance A from equilibrium?