Answer to Essential Question 12.4: Absolutely. All aspects of the motion of the oscillating block match one component of the motion of the object experiencing uniform circular motion. If the position of the block is given by $\vec{x} = A\cos(\omega t)$, then its velocity and acceleration are given by:

$\vec{v}_{x,disk1} = \vec{v}_{block1} = -v\sin(\omega t) = -A\omega\sin(\omega t)$ and $\vec{a}_{x,disk1} = \vec{a}_{block1} = -\frac{v^2}{A}\cos(\omega t) = -A\omega^2\cos(\omega t)$.

Here, v represents the constant speed of disk 1 as it moves in uniform circular motion.

12-5 Hallmarks of Simple Harmonic Motion

Simple harmonic motion (often referred to as SHM) is a special case of oscillatory motion. An object oscillating in one dimension on an ideal spring is a prime example of SHM. The characteristics of simple harmonic motion include:

- A force (and therefore an acceleration) that is opposite in direction, and proportional to, the displacement of the system from equilibrium. Such a force, that acts to restore the system to equilibrium, is known as a **restoring force**.
- No loss of mechanical energy.
- An angular frequency ω that depends on properties of the system.
- Position, velocity, and acceleration given by Equations 12.3 – 12.5:

$\vec{x} = A\cos(\omega t)$. (Equation 12.3: **Position in simple harmonic motion**)

$\vec{v} = -v_{\max}\sin(\omega t) = -A\omega\sin(\omega t)$. (Equation 12.4: **Velocity in SHM**)

$\vec{a} = -a_{\max}\cos(\omega t) = -\frac{v^2}{A}\cos(\omega t) = -A\omega^2\cos(\omega t)$. (Eq. 12.5: **Acceleration in SHM**)

The above equations apply if the object is released from rest from $\vec{x} = +A$ at $t = 0$. Starting the block with different initial conditions requires a modification of the equations.

Combining Equations 12.3 and 12.5, in any simple harmonic motion system we see that the acceleration is opposite in direction, and proportional to, the displacement:

$\vec{a} = -\omega^2\,\vec{x}$. (Equation 12.6: **Connecting acceleration and displacement in SHM**)

In general, the angular frequency (ω), frequency (f), and period (T) are connected by:

$\omega = 2\pi f = \frac{2\pi}{T}$. (Eq. 12.7: **Relating angular frequency, frequency, and period**)

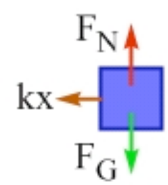

Figure 12.12: The free-body diagram of a block connected to a spring of spring constant k. The block is displaced to the right of the equilibrium point by a distance x.

What determines the angular frequency ω in a particular situation? Let's return to the free-body diagram of a block on a spring, shown in Figure 12.12.

Applying Newton's Second Law horizontally, $\sum \vec{F}_x = m\vec{a}$, we get:

$-k\,\vec{x} = m\vec{a}$.

Re-arranging gives $\vec{a} = -(k/m)\vec{x}$. Comparing this result to the general SHM Equation 12.6 tells us that, for a mass on an ideal spring, $\omega^2 = k/m$, or:

$\omega = \sqrt{\frac{k}{m}}$. (Equation 12.8: **Angular frequency for a mass on a spring**).

This is a typical result, that the angular frequency is given by the square root of a parameter related to the restoring force (or torque, in rotational motion) divided by the inertia.

EXAMPLE 12.5 – Plotting graphs of position, velocity, and acceleration versus time

Once again, let's attach a block to a spring and release the block from rest from a position $\vec{x} = +A$ (relative to $\vec{x} = 0$, which is the equilibrium position). The block oscillates back and forth with a period of $T = 4.00$ s.

(a) Plot graphs of the block's position, velocity, and acceleration as a function of time over two complete oscillations.

(b) Compare the position graph to the velocity graph.

(c) How does the acceleration graph compare to the position graph?

SOLUTION

(a) We can make use of Equations 12.3 – 12.5 to plot the graphs. Before doing so, we can solve for the angular velocity ω, using:

$$\omega = \frac{2\pi}{T} = \frac{2\pi \text{ rad}}{4.00 \text{ s}} = 1.57 \text{ rad/s} .$$

Also, it makes it easier to plot the graphs if we remember that, if the block is released from rest, it returns to its starting point after one period; after half a period it comes instantaneously to rest on the far side of equilibrium; and at times of T/4 and 3T/4 it is passing through equilibrium at its maximum speed. Determining when each graph passes through zero, when it reaches its largest positive and negative values, and then connecting these points with sinusoidally oscillating graphs, gives the results shown in Figure 12.13.

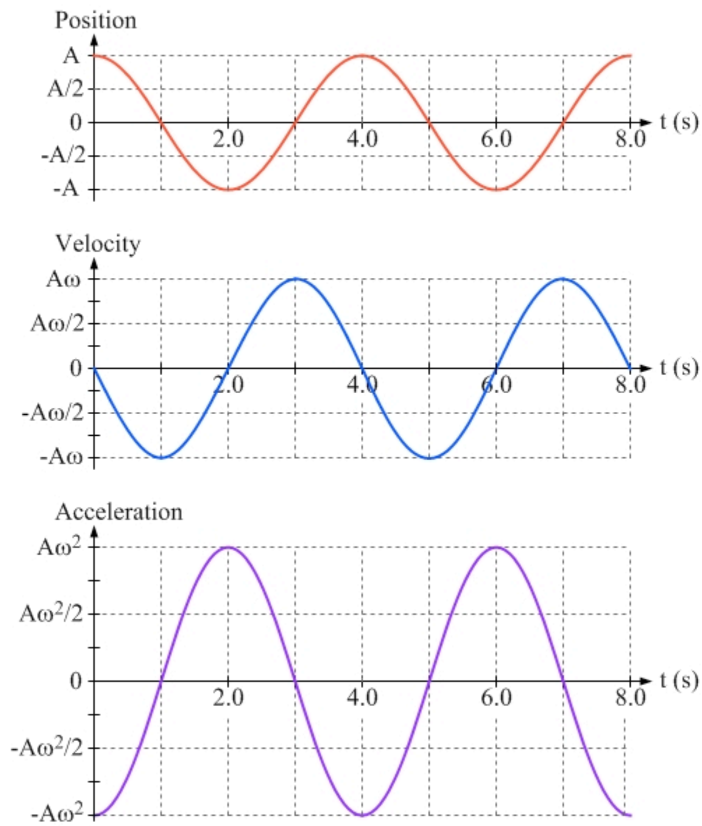

Figure 12.13: Graphs of the position, velocity, and acceleration, as a function of time, of the block in Example 12.5 for two complete oscillations of the block.

(b) Comparing the position and velocity graphs in Figure 12.13, we can see that the block's speed is maximum when the block's displacement from equilibrium is zero. Conversely, the block's speed is zero when the magnitude of the block's displacement from equilibrium is maximized. These observations are consistent with what is taking place with the energy. The kinetic energy is proportional to the speed squared and the elastic potential energy is proportional to the square of the magnitude of the displacement from equilibrium. Kinetic energy is maximum when the elastic potential energy is zero, and vice versa.

(c) Comparing the position and acceleration graphs, we see that one is the opposite of the other, in the sense that when the position is positive the acceleration is negative, and vice versa. This is expected because one of the hallmarks of simple harmonic motion is that $\vec{a} = -\omega^2 \vec{x}$.

Related End-of-Chapter Exercises: 32, 41, 46.

Essential Question 12.5: Return to the situation described in Example 12.5, but now increase the angular frequency by a factor of 2. We can accomplish this by either changing only the spring constant or by changing only the mass. Can we tell which one was changed by looking at the resulting graphs of position, velocity, and/or acceleration as a function of time? Assume that the block is released from rest from the same point it was in Example 12.5, and that the equilibrium position remains the same.