Answer to Essential Question 12.3: In fact, this result is generally true. As long as the spring is ideal, then ***the time it takes a block to move through one complete oscillation is independent of the amplitude of the oscillation***. The **amplitude** is defined as the maximum distance an object gets from its equilibrium position during its oscillatory motion.

12-4 The Connection with Circular Motion

So far we have looked at how to apply force and energy ideas to springs. Let's now explore an interesting connection between what is called **simple harmonic motion** (oscillatory motion without any loss of mechanical energy), and uniform circular motion.

EXPLORATION 12.4 – Connecting circular motion to simple harmonic motion

Take the two spring-block systems we investigated at the end of the previous section and place them beside a large turntable that is rotating about a vertical axis. Set the constant angular speed of the turntable so that the turntable undergoes one complete revolution in the time it takes the blocks on the springs to move through one complete oscillation. As shown in Figure 12.8, there are two disks on the turntable, one a distance A from the center and the other a distance $2A$ from the center. The blocks are simultaneously released from rest at the instant the disks pass through the position shown in the figure.

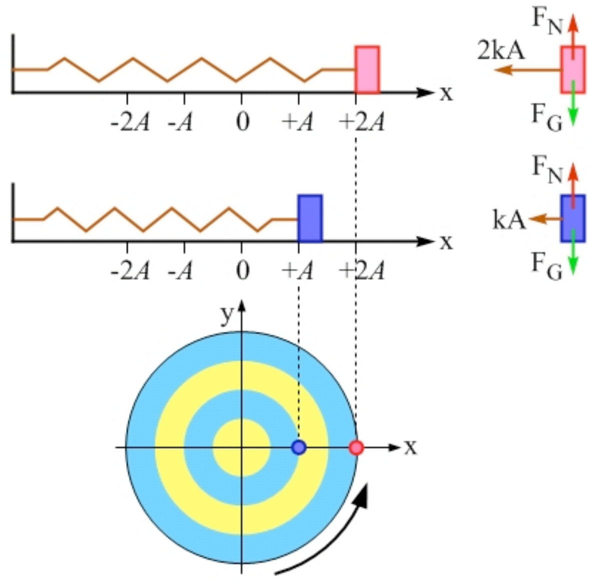
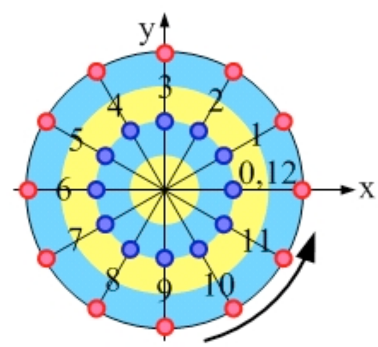


Figure 12.8: Comparing systems – two disks on a rotating turntable and two oscillating block-and-spring systems.

Another amazing thing happens. As the disks spin at constant angular velocity and the blocks oscillate back and forth, the motion of block 1 matches the motion of disk 1, while the motion of block 2 matches the motion of disk 2. The position of the left-hand side of each block ***is at all times*** equal to the x-coordinate of the position of the center of its corresponding disk, taking the origin to be at the center of the turntable and using the x-y coordinate system shown in Figure 12.8.

Step 1 – ***Sketch two separate motion diagrams, one showing the successive positions of disk 1 and the other showing the successive positions of disk 2, as the turntable undergoes one complete revolution. Plot the positions at regular time intervals which, because the disk rotates at a constant rate, correspond to regular angular displacements.*** Motion diagrams for the disks are shown in Figure 12.9, showing positions at 30° intervals.



Step 2 – ***Now add motion diagrams for the two blocks, sketching their positions so they agree with the statement above, that the left-hand side of each block is at all times equal to the x-coordinate of the position of the center of its corresponding disk.*** These motion diagrams are shown in Figure 12.10.

Figure 12.9: Motion diagrams for the two disks, showing their positions at 30° intervals. Because the turntable (and each disk) rotates at a constant rate, these equal angular displacements correspond to the equal time intervals we're used to seeing on motion diagrams.

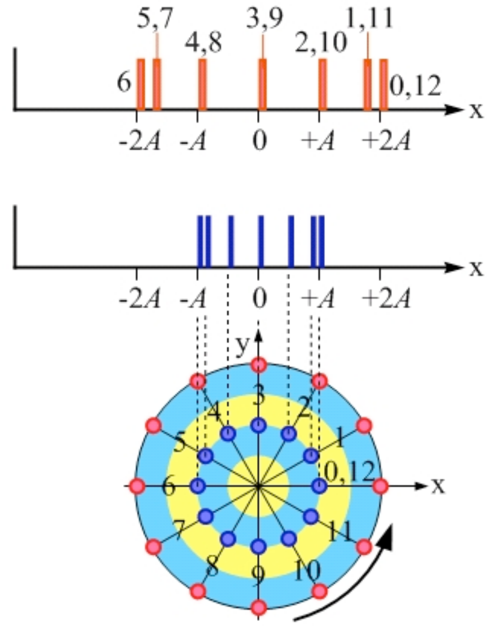


Figure 12.10: Motion diagrams for the two blocks, showing that the motion of a block experiencing simple harmonic motion exactly matches the motion of a well-chosen object experiencing uniform circular motion. The springs have been removed from the picture for clarity, and the motion diagrams for the blocks show the successive positions of the left-hand side of each block during its motion. The motion of a block matches the *x*-component of the motion of the corresponding disk on the turntable.

Step 3 – ***Measuring angles counterclockwise from the positive x-axis, write an equation giving the x-coordinate of disk 1 as a function of time. Hint: first write out the x-coordinate in terms of an arbitrary angle the turntable has rotated through, and then express that angle in terms of time and the turntable's constant angular speed*** $\boldsymbol{\omega}$**.** Figure 12.11 shows the position of the disk 1 when the turntable has rotated through some arbitrary angle θ from its initial position. Its *x*-position at this angle can be found from the adjacent side of the right-angled triangle: $\vec{x} = A\cos(\theta)$. Since the angular velocity is constant, however, and the initial angle $\theta_i = 0$ we can express the angle as: $\theta = \theta_i + \omega t = 0 + \omega t = \omega t$. Substituting this into our expression for the disk's x-position gives: $\vec{x} = A\cos(\omega t)$.

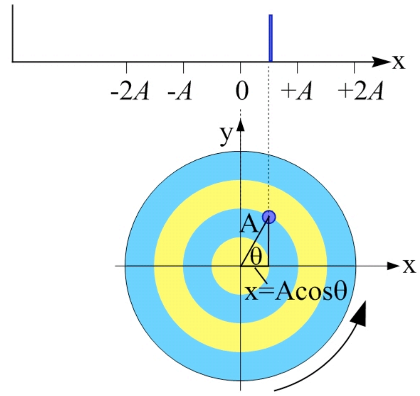


Figure 12.11: The position of block 1 and disk 1 after the turntable has rotated through an arbitrary angle θ.

Step 4 – ***Based on the results above, what is the equation giving the x-position of block 1 (actually, the position of the left edge of block 1) as a function of time? What is the equation giving the position of block 2 as a function of time?*** Because the motion of block 1 matches exactly the *x*-component of the motion of disk 1, the equation that gives the disk's *x*-position must also gives the block's *x*-position. Thus, for block 1 we have:

$$\vec{x} = A\cos(\omega t) \quad . \qquad \text{(Eq. 12.3: \textbf{Position-versus-time for simple harmonic motion})}$$

Using the convention introduced earlier in this book, in which a + or – sign is used to represent the direction of a vector in one dimension, the right-hand side of equation 12.3 can be viewed as a vector quantity, with the sign hidden in the cosine. We get a positive sign for some values of time and a negative sign for others. The equation for block 2 is virtually identical to that of block 1, with the only change being the extra factor of 2. For block 2: $\vec{x} = 2A\cos(\omega t)$.

Key ideas: There is an interesting connection between simple harmonic motion and uniform circular motion. One-dimensional simple harmonic motion matches one component of a carefully chosen two-dimensional uniform circular motion. This allows us to write an equation of motion for an object experiencing simple harmonic motion: $\vec{x} = A\cos(\omega t)$. In this context, ω is known as the angular frequency. **Related End-of-Chapter Exercises: 42, 43.**

Essential Question 12.4: We showed above how the position of a block oscillating on a spring matches one component of the position of an object experiencing uniform circular motion. Can we make similar conclusions about the velocity and acceleration of the block on the spring?