

**Answer to Essential Question 12.2:** To answer this question, we can use the fact that the elastic potential energy is proportional to  $x^2$ . Doubling  $x$ , the distance from equilibrium, increases the elastic potential energy by a factor of 4. Thus, the elastic potential energy is 6 J when  $x = 10$  cm.

### 12-3 An Example Involving Springs and Energy

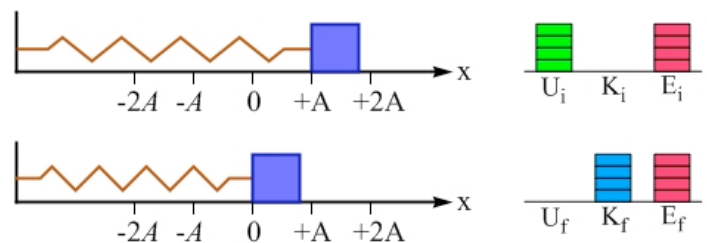
#### EXAMPLE 12.3 – A fast-moving block

(a) A block of mass  $m$ , which rests on a horizontal frictionless surface, is attached to an ideal horizontal spring. The block is released from rest when the spring is stretched by a distance  $A$  from its natural length. What is the block's maximum speed during the ensuing oscillations?

(b) If the block is released from rest when the spring is stretched by  $2A$  instead, how does the block's maximum speed change?

#### SOLUTION

(a) Let's begin, as usual, with a diagram of the situation (see Figure 12.6). When will the block achieve its maximum speed? Maximum speed corresponds to maximum kinetic energy, which corresponds to minimum potential energy. The gravitational potential energy is constant, since there is no up or down motion, so we can focus on the elastic potential energy. The elastic potential energy is a minimum (zero, in fact) when the block passes through equilibrium, where the spring is at its natural length. Energy bar graphs for the two points are shown in Figure 12.6.



**Figure 12.6:** Diagrams of the block at the release point and at the equilibrium position, and the corresponding energy bar graphs.

Let's continue with the energy analysis by writing out the conservation of energy equation:  $K_i + U_i + W_{nc} = K_f + U_f$ . The initial point is the point from which the block is released, while the final point is the equilibrium position.

$K_i = 0$ , because the block is released from rest from the initial point.

$W_{nc} = 0$ , because there is no work being done by non-conservative forces.

We can neglect gravitational potential energy, because there is no vertical motion. This gives  $U_f = 0$ , because the elastic potential energy is also zero at the final point.

We have thus reduced the energy equation to:  $U_{ei} = K_f$ . This gives:

$$\frac{1}{2}kA^2 = \frac{1}{2}mv_{\max}^2. \text{ Solving for the maximum speed gives: } v_{\max} = A\sqrt{\frac{k}{m}}.$$

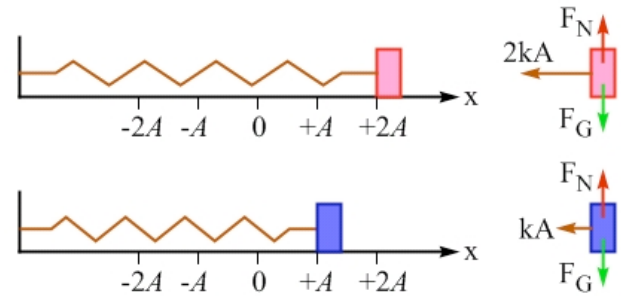
Is this answer reasonable? The maximum speed is larger if we start the block farther from equilibrium (where the spring exerts a larger force); if we increase the spring constant (also increasing the force); or if we decrease the mass (increasing acceleration). This all makes sense.

(b) If we start the block from  $2A$  away from equilibrium, we simply replace  $A$  in our equation above by  $2A$ , showing us that the maximum speed is twice as large:

$$v'_{\max} = 2A\sqrt{\frac{k}{m}}.$$

**Related End-of-Chapter Exercises: 4, 5.**

We can make an interesting generalization based on further analysis of the situation in Example 12.3. Take two blocks, one red and one blue but otherwise identical, and two identical springs. Attach each block to one of the springs, and place these two block-spring systems on frictionless horizontal surfaces. As shown in Figure 12.7, we will release one block from rest from a distance  $A$  from equilibrium and the other from a distance  $2A$  from equilibrium. If the blocks are released simultaneously, which block reaches the equilibrium point first?



**Figure 12.7:** Identical blocks attached to identical springs. The blocks are released from rest simultaneously. Block 2, at the top, is released from a distance  $2A$  from equilibrium. Block 1 is released from a distance  $A$  from equilibrium. The initial free-body diagrams are also shown.

Block 2 has an initial acceleration twice as large as that of block 1, because block 2 experiences a net force that is twice as large as that experienced by block 1. The accelerations steadily decrease, because the spring force decreases as the blocks get closer to equilibrium, but we can neglect this change if we choose a time interval that is sufficiently small.

At the end of this time interval,  $\Delta t$ , what is the speed of each block? We're choosing a small time interval so that we can apply a constant-acceleration analysis. Remembering that the blocks are released from rest, so  $v_i = 0$ , we have:

$$\begin{aligned} \text{for block 1,} \quad \bar{v}_1 &= \bar{v}_{i1} + \bar{a}_1 \Delta t = \bar{a}_1 \Delta t ; \\ \text{for block 2,} \quad \bar{v}_2 &= \bar{v}_{i2} + \bar{a}_2 \Delta t = \bar{a}_2 \Delta t = 2\bar{a}_1 \Delta t = 2\bar{v}_1 . \end{aligned}$$

What about the distance each block travels? Here we can apply another constant acceleration equation:

$$\begin{aligned} \text{for block 1} \quad \Delta \bar{x}_1 &= \bar{v}_{i1} \Delta t + \frac{1}{2} \bar{a}_1 (\Delta t)^2 = \frac{1}{2} \bar{a}_1 (\Delta t)^2 ; \\ \text{for block 2} \quad \Delta \bar{x}_2 &= \bar{v}_{i2} \Delta t + \frac{1}{2} \bar{a}_2 (\Delta t)^2 = \frac{1}{2} \bar{a}_2 (\Delta t)^2 = \frac{1}{2} (2\bar{a}_1) (\Delta t)^2 = 2\Delta \bar{x}_1 . \end{aligned}$$

At the end of the time interval, block 1 is  $A - \Delta x_1$  from equilibrium and block 2 is exactly twice as far from equilibrium as block 1, at  $2A - 2\Delta x_1 = 2(A - \Delta x_1)$  from equilibrium. Thus, after this small time interval has passed, block 2 is still twice as far from equilibrium as block 1, its velocity is twice as large, and its acceleration is twice as large. We could keep the process going, following the two blocks as time goes by, and we would find this always to be true, that block 2's velocity, acceleration, and displacement from equilibrium, is always double that of block 1. This is true at all times, even after the blocks pass through their equilibrium positions to the far side of equilibrium.

This leads to an amazing conclusion – **that the two blocks take exactly the same time to reach equilibrium** (and to complete one full cycle of an oscillation). This is because block 2 experiences twice the displacement of block 1, but its average velocity is also twice as large. Because the time is the distance divided by the average velocity, these factors of two cancel out.

**Essential Question 12.3:** Above we analyzed the situation of two identical (aside from color) blocks, oscillating on identical springs, and found the time to reach equilibrium (or to complete one full oscillation) to be the same. Was that just a coincidence that happened to work out because the starting displacements from equilibrium were in a 2:1 ratio, or can we generalize and say that the time is the same no matter where the block is released?