Answer to Essential Question 12.1: This estimated time is less than the actual time. The closer the block gets to the equilibrium position, the smaller the force that is exerted on it by the spring, and the smaller the magnitude of the block's acceleration. Because the block generally has a smaller acceleration than the acceleration we used in the constant-acceleration analysis, it will take longer to reach equilibrium than the time we calculated with the constant-acceleration analysis. Thus, remember not to use constant-acceleration equations in harmonic motion situations! We'll learn how to calculate exact times in sections 12-4 to 12-6.

12-2 Springs and Energy Conservation

Now that we have seen how to incorporate springs into a force perspective, let's go on to consider how to fit springs into what we know about energy.

EXPLORATION 12.2 – Another kind of potential energy

Step 1 – ***Attach a block to a spring, and position the block so that the spring is stretched. Let's neglect friction, so when you release the block from rest it oscillates back and forth about the equilibrium position. What is going on with the energy of the system as the block oscillates?*** As the block oscillates, its speed increases from zero to some maximum value, then decreases to zero again, and keeps doing this over and over. The kinetic energy of the system does exactly the same thing, since it is proportional to the square of this speed. Where does the energy go when the kinetic energy decreases, and where does it comes from when the kinetic energy increases?

The energy is stored as potential energy in the spring. This is similar to what happens when we throw a ball up into the air. As the ball rises, the ball's loss of kinetic energy is offset by the gain in the gravitational potential energy of the Earth-ball system, and then that potential energy is transformed back into kinetic energy. Compressed or stretched springs also store potential energy. Such energy is known as **elastic potential energy**.

Step 2 - ***Consider the graph of force, as a function of the displacement of the end of the spring, shown in Figure 12.4. As we did in Chapter 6, defining the change in gravitational potential energy to be the negative of the work done by gravity on an object, find an expression for the change in elastic potential energy as the end of the spring is displaced from its equilibrium position (x = 0) to some arbitrary final position x. Make use of the fact that work is the area under the force-versus-position graph in Figure 12.4.***

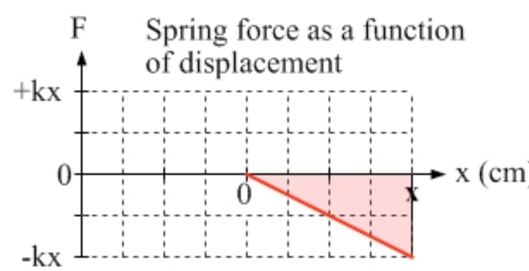

Figure 12.4: The work done by a spring when its end is displaced from the equilibrium position to a point x away from equilibrium is represented by the shaded area in the graph.

The area in question is that of the right-angled triangle shown in Figure 12.4. The area is negative because the force is negative the entire time. The area under the curve is given by:

$$\text{area} = -\frac{1}{2}\text{base}\times\text{height} = -\frac{1}{2}x(kx) = -\frac{1}{2}kx^2 .$$

This area represents the work done by the spring. This work is negative because the spring force is opposite in direction to the displacement. Because ΔU_e, the change in the elastic potential energy, is the negative of the work, we have $\Delta U_e = \frac{1}{2}kx^2$ in this case.

Step 3 – ***How much elastic potential energy is stored in the spring when the spring is at its natural length?*** None. If we attach a block to such a spring and release the block from rest, no motion occurs because the system is at equilibrium. There is no transformation of elastic potential energy into kinetic energy because the system has no elastic potential energy when the spring is at its natural length – the equilibrium position is the zero for elastic potential energy.

Step 4 – ***Combine the results from parts 2 and 3 to determine the expression for the elastic potential energy stored in a spring when the end of the spring is displaced a distance x from its equilibrium position.*** In step 3 we found the change in elastic potential energy in displacing the end of the spring from its equilibrium position to a point *x* away from equilibrium to be $\Delta U_e = \frac{1}{2}kx^2$. This change in elastic potential energy is equal to the final elastic potential energy minus the initial elastic potential energy. However, we found the initial elastic potential energy to be zero in step 3, which means the expression for elastic potential energy is simply:

$$U_e = \frac{1}{2}kx^2 . \qquad \text{(Equation 12.2: \textbf{Elastic potential energy})}$$

Key ideas: Compressed or stretched springs store energy – this is known as elastic potential energy. For an ideal spring, the elastic potential energy is $U_e = \frac{1}{2}kx^2$.

Related End-of-Chapter Exercises: 9, 48.

Now that we know the form of the elastic potential energy equation, we can incorporate springs into the conservation of energy equation we first used in chapter 7:

$$K_i + U_i + W_{nc} = K_f + U_f . \qquad \text{(Equation 7.1)}$$

Graphs of the energies as a function of position are interesting. Consider a block attached to a spring. The block is oscillating back and forth on a frictionless surface, so the total mechanical energy stays constant. An easy way to graph the kinetic energy is to exploit energy conservation, $E = K + U$. Solving for the kinetic energy as a function of position gives:

$$K = E - U = E - \frac{1}{2}kx^2 .$$

Graphs of the energies as a function of position are shown in Figure 12.5, for a situation in which the total mechanical energy is 4.0 J. After tracing out the complete energy curves over half an oscillation, the system re-traces these energy-versus-position plots as the block oscillates.

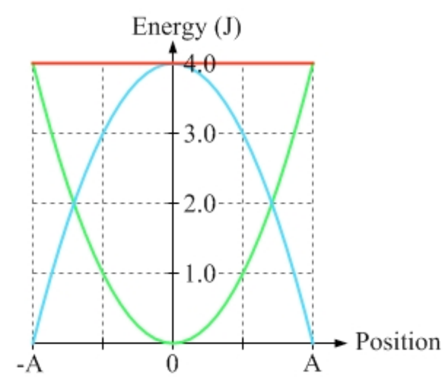

Figure 12.5: Graphs of the system's kinetic energy (zero at -A and A), elastic potential energy (zero at x = 0), and total mechanical energy (constant), as a function of position. The system traces over each of the energy graphs every half oscillation.

Essential Question 12.2: Consider a system consisting of a block attached to an ideal spring. The block is oscillating on a horizontal frictionless surface. When the block is 20 cm away from the equilibrium position, the elastic potential energy stored in the spring is 24 J. What is the elastic potential energy when the block is 10 cm away from equilibrium?