12-1 Hooke's Law

We probably all have some experience with springs. One observation we can make is that it doesn't take much force to stretch or compress a spring a small amount, but the more we try to compress or stretch it, the more force we need. We'll use a model of an ideal spring, in which the magnitude of the force associated with stretching or compressing the spring is proportional to the distance the spring is stretched or compressed.

The equation describing the proportionality of the spring force with the displacement of the end of the spring from its natural length is known as Hooke's law.

$$\vec{F}_{Spring} = -k\,\vec{x}\ .$$ (Equation 12.1: **Hooke's Law**)

The negative sign is associated with the restoring nature of the force. When you displace the end of the spring in one direction from its equilibrium position, the spring applies a force in the opposite direction, essentially in an attempt to return the system toward the equilibrium position (the position where the spring is at its natural length, neither stretched nor compressed). The force applied by the spring is proportional to the distance the spring is stretched or compressed relative to its natural length.

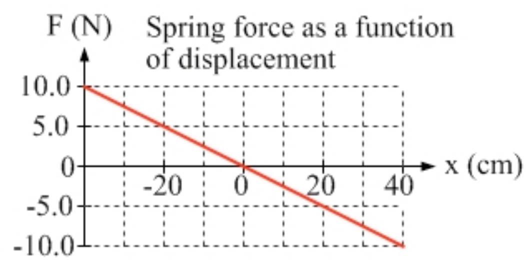


Figure 12.1: A graph of the force applied by a particular spring as a function of the displacement of the end of the spring from its equilibrium position.

The k in the Hooke's law equation is known as the **spring constant**. This is a measure of the stiffness of the spring. If you have two different springs and you stretch them the same amount from equilibrium. The one that requires more force to maintain that stretch has the larger spring constant. Figure 12.1 shows the Hooke's law relationship as a graph of force as a function of the amount of compression or stretch of a particular spring from its natural length.

The Hooke's law relationship is illustrated in Figure 12.2, where $x = 0$ means the spring is neither stretched nor compressed from its natural length. A block attached to spring has been released and is oscillating on a frictionless surface. Free-body diagrams are shown in Figure 12.2, illustrating how the force exerted by the spring on the block depends on the displacement of the end of the spring from its equilibrium position.

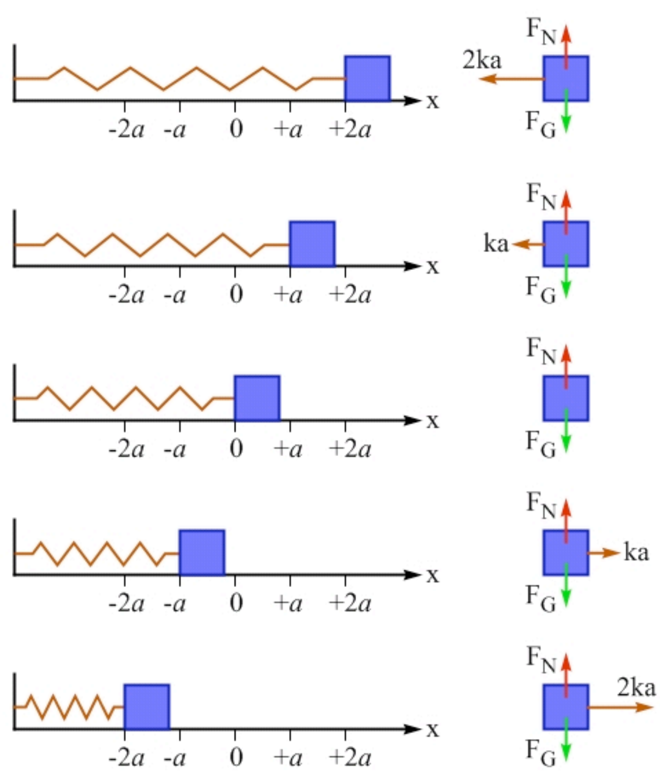


Figure 12.2: A block attached to an ideal spring oscillates on a frictionless surface. By looking at the free-body diagrams of the block when the block is at various positions, we can see that the force applied by the spring on the block is proportional to the displacement of the end of the spring from its equilibrium position, and opposite in direction to that displacement.

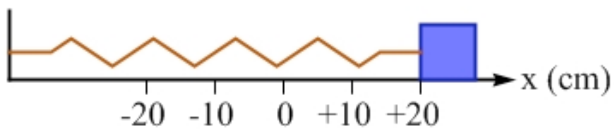
EXAMPLE 12.1 – Initial acceleration of a block

A block of mass 300 g is attached to a horizontal spring that has a spring constant of 6.0 N/m. The block is on a horizontal frictionless surface. You release the block from rest when the spring is stretched by 20 cm.

(a) Sketch a diagram of the situation, and a free-body diagram of the block immediately after you release the block.
(b) Determine the block's initial acceleration.
(c) What happens to the block's free-body diagram as the block moves to the left?

SOLUTION

(a) The diagram and free-body diagram are shown in Figure 12.3. After you release the block, only three forces act on the block. The downward force of gravity is balanced by the upward normal force applied by the surface. The third force is the force applied by the spring. The spring force is directed to the left because the end of the spring has been displaced to the right from its equilibrium position.



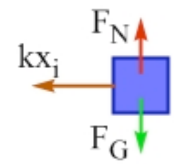


Figure 12.3: A diagram of the block and spring, and the free-body diagram of the block, immediately after the block is released from rest.

(b) Here we can apply Newton's Second Law horizontally, $\sum \vec{F}_x = m\vec{a}$, taking right to be the positive x-direction. This gives: $-F_{Spring} = m\vec{a}$.

Now we can bring in Equation 12.1, $\vec{F}_{Spring} = -k\,\vec{x}$, to get: $-k\,x_i = m\vec{a}_i$.

Note that we use only one minus sign in the equation because we're substituting for the magnitude of the spring force only. The one minus sign represents the direction of the spring force, which is to the left. Solving for the block's initial acceleration gives:

$$\vec{a}_i = +\frac{k\,x_i}{m} = -\frac{(6.0\,\mathrm{N/m})(0.20\,\mathrm{m})}{0.30\,\mathrm{kg}} = -4.0\,\mathrm{N/kg}.$$

The initial acceleration is 4.0 N/kg to the left.

(c) As the block moves to the left, nothing changes about the vertical forces, but the spring force steadily decreases in magnitude because the stretch of the spring steadily decreases. Once the block goes past the equilibrium position, the spring force points to the right, and increases in magnitude as the compression increases. The dependence of the spring force on the block's position is shown, for five different positions, in Figure 12.2.

Related End-of-Chapter Exercises: 16, 55.

Essential Question 12.1: Let's say we estimated the time it takes the block in Example 12.1 to reach equilibrium, by assuming the block's acceleration is constant at 4.0 N/kg to the left. Is our estimated time smaller than or larger than the time it actually takes the block to reach the equilibrium point?