Answer to Essential Question 12.7: The two free-body diagrams are shown in Figure 12.19. When the ball is at rest its acceleration is zero. Applying Newton's Second Law tells us that, in this case, the force of tension exactly balances the force of gravity, so $F_T = mg$. When the ball is oscillating it is moving along a circular arc as it passes through the equilibrium position. In this case there is a non-zero acceleration, the centripetal acceleration directed toward the center of the circular arc. To produce the upward acceleration the upward force of tension must be larger than the downward force of gravity. Applying Newton's Second Law shows that the force of tension increases to:

$$F_T = mg + m\frac{v^2}{L} .$$

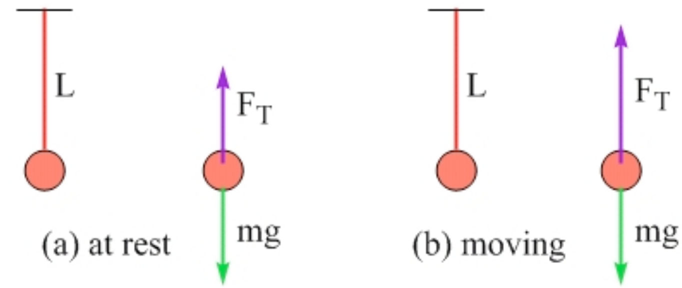

Figure 12.19: Free-body diagrams for the ball on the string when it is (a) at rest, and (b) passing through equilibrium with a speed *v*.

Chapter Summary

Essential Idea

Harmonic oscillations are important in many applications, from musical instruments to clocks and, in the human body, from walking to the creation of sounds with our vocal cords. Even though we have focused on two basic models in this chapter, the block on the spring and the simple pendulum, the same principles apply in many real-life situations.

Springs

An ideal spring obeys Hooke's Law, $\vec{F}_{Spring} = -k\,\vec{x}$. (Equation 12.1: **Hooke's Law**)

k is the spring constant, a measure of the stiffness of the spring.

Springs that are stretched or compressed store energy. This energy is known as elastic potential energy.

$$U_e = \frac{1}{2}kx^2 .$$ (Equation 12.2: **Elastic potential energy for an ideal spring**)

When an object oscillates on a spring, the angular frequency of the oscillations depends on the mass of the object and the spring constant of the spring.

$$\omega = \sqrt{\frac{k}{m}} .$$ (Equation 12.8: **The angular frequency of a mass on a spring**)

Simple Harmonic Motion and Energy Conservation

Energy conservation is a useful tool for analyzing oscillating systems. When springs are involved we use elastic potential energy, an idea introduced in this chapter. To analyze a pendulum in terms of energy conservation nothing new whatsoever is needed.

Hallmarks of Simple Harmonic Motion

The main features of a system that undergoes simple harmonic motion include:

- No loss of mechanical energy.
- A restoring force or torque that is proportional to, and opposite in direction to, the displacement of the system from equilibrium.

In this situation the acceleration of the system is related to its position by:

$\vec{a} = -\omega^2 \vec{x}$, (Eq. 12.6: **The connection between acceleration and displacement**)

where the angular frequency ω is generally given by the square root of some elastic property of the system (such as the spring constant) divided by an inertial property (such as the mass).

Time and Simple Harmonic Motion

When we are interested in how a simple harmonic oscillator evolves over time the following equations are extremely useful. These were derived by looking at the connection between simple harmonic motion and one component of the motion of an object experiencing uniform circular motion.

$\vec{x} = A\cos(\omega t)$. (Equation 12.3: **Position in simple harmonic motion**)

$\vec{v} = -A\omega\sin(\omega t)$. (Equation 12.4: **Velocity in simple harmonic motion**)

$\vec{a} = -A\omega^2\cos(\omega t)$. (Eq. 12.5: **Acceleration in simple harmonic motion**)

Equations 12.3 – 12.5 apply when the object is released from rest at $t = 0$ from a distance A from equilibrium.

In general, the angular frequency (ω), frequency (f), and period (T) are connected by:

$\omega = 2\pi f = \frac{2\pi}{T}$. (Eq. 12.7: **Relating angular frequency, frequency, and period**)

The Simple Pendulum

A simple pendulum, consisting of an object on the end of a string, is another good example of an oscillating system. As long as the amplitude of the oscillations is small (less than about 10°) and mechanical energy is conserved then the motion is simple harmonic. For larger angles the motion diverges from simple harmonic because the restoring torque is not directly proportional to the angular displacement.

$\omega = \sqrt{\frac{g}{L}}$. (Equation 12.11: **Angular frequency of a simple pendulum**)