End-of-Chapter Exercises

Exercises 1 – 12 are conceptual questions that are designed to see if you have understood the main concepts of the chapter.

1. When a spring is compressed 10 cm, compared to its natural length, the spring exerts a force of 5 N. What is the spring force when the spring is stretched by 10 cm compared to its natural length, instead?

2. A block on a horizontal surface is attached to an ideal horizontal spring, as shown in Figure 12.20. When the block compresses the spring by 10 cm, the spring exerts a force of 10 N on the block. The block is then moved either left or right to a new position, where the force the spring exerts on the block has a magnitude of 20 N. How far has the block been moved? State all possible answers.

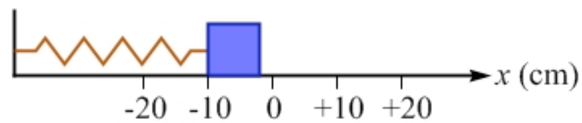

Figure 12.20: A block connected to an ideal horizontal spring. The block initially compresses the spring by 10 cm. For Exercises 2 and 3.

3. A block on a frictionless horizontal surface is attached to an ideal horizontal spring, as shown in Figure 12.20. When the block compresses the spring by 10 cm, the elastic potential energy stored in the spring is 10 J. The block is then moved either left or right to a new position, where the elastic potential energy stored in the spring is 40 J. How far has the block been moved? State all possible answers.

4. A small ball is loaded into a spring gun, compressing the spring by a distance A. When the trigger is pressed, the ball emerges from the gun with a speed v. The ball is loaded into the gun again, this time compressing the spring by a distance $2A$. With what speed will the ball emerge from the gun this time? Justify your answer.

5. A block is attached to a spring and the system is placed on a horizontal frictionless surface with the other end of the spring anchored firmly to a wall. The block is then displaced from equilibrium until 8.0 J of elastic potential energy has been stored in the spring. The block is then released from rest and the block oscillates back and forth about the equilibrium position. Sketch energy bar graphs for this system, showing the elastic potential energy, kinetic energy, and total mechanical energy when the system is (a) at the point where it is released from rest; (b) halfway between that point and the equilibrium position; (c) at the equilibrium position.

6. A block on a spring experiences simple harmonic motion with amplitude A and period T. For one complete oscillation, determine (a) the block's displacement; (b) the total distance traveled by the block; (c) the block's average velocity; (d) the block's average speed; (e) the block's average acceleration.

7. Consider the following four cases. In each case, the block experiences simple harmonic motion of amplitude A.
   Case 1: a block of mass m connected to a spring of spring constant k.
   Case 2: a block of mass m connected to a spring of spring constant $2k$.
   Case 3: a block of mass $2m$ connected to a spring of spring constant k.
   Case 4: a block of mass $2m$ connected to a spring of spring constant $2k$.
   Rank these cases, from largest to smallest, based on (a) their angular frequency; (b) their total mechanical energy; (c) the maximum speed reached by the block during its motion. Your answers should have a form similar to 3>1>2=4.

8. You have three blocks, of mass m, $2m$, and $3m$, and three springs of spring constant k, $2k$, and $3k$. You can attach any one of the blocks to any one of the springs, displace the block from equilibrium by a distance of A, $2A$, or $3A$, and release the block from rest so it experiences simple harmonic motion. Which combination of these three parameters (mass, spring constant, and amplitude) results in oscillations with the largest (a) angular frequency? (b) period? (c) total mechanical energy? (d) speed when the block passes through equilibrium?

9. A block of mass m is connected to a spring with a spring constant k, displaced a distance A from equilibrium. Upon being released from rest, the block experiences simple harmonic motion. Let's say you wanted to double the mechanical energy of the system. (a) Could you accomplish this by changing the mass, but keeping everything else the same? If so, what would the new mass be? (b) Could you accomplish this by changing the spring constant, but keeping everything else the same? If so, what would the new spring constant be? (c) Could you accomplish this by changing the amplitude of the oscillation, but keeping everything else the same? If so, what would the new amplitude be?

10. Repeat Exercise 9, except this time you want to double the angular frequency instead of the energy.

11. Consider the following four simple pendula. In each case, the pendulum experiences simple harmonic motion with a maximum angular displacement θ_{max} , where θ_{max} is small enough that the small-angle approximation can be used.
Case 1: a pendulum consisting of a ball of mass m on a string of length L.
Case 2: a pendulum consisting of a ball of mass m on a string of length $2L$.
Case 3: a pendulum consisting of a ball of mass $2m$ on a string of length L.
Case 4: a pendulum consisting of a ball of mass $2m$ on a string of length $2L$.
Rank these cases, from largest to smallest, based on (a) their angular frequency; (b) their total mechanical energy; (c) the maximum speed reached by the block during its motion. Your answers should have the form 3>1>2=4.

12. Return to Exercise 11. Now rank the cases, from largest to smallest, based on the tension in the string when the ball passes through the equilibrium position.

Exercises 13 – 17 deal with various situations involving ideal springs.

13. A spring hangs vertically down from a support, with a ball with a weight of 6.00 N hanging from the spring's lower end. (a) If the ball remains at rest and the spring is stretched by 20.0 cm with respect to its natural length, what is the spring constant of the spring?

14. Consider again the situation described in Exercise 13. You now take the spring, cut it in half, and hang the same ball from one half of the spring (the other half you don't use at all) so the ball again remains at rest as it hangs vertically from the spring. (a) How much is the spring stretched from its natural length? Briefly justify your answer. (b) How does the spring constant of this new spring compare to the spring constant of the original spring?

15. A small ball with a mass of 50 g is loaded into a spring gun, compressing the spring by 12 cm. When the trigger is pressed, the ball emerges horizontally from the barrel at a height of 1.4 m above the floor. It then strikes the floor after traveling a horizontal distance of 2.5 m. Use $g = 9.8$ m/s^2. Assuming all the energy stored in the spring is transferred to the ball, determine the spring constant of the spring.

16. A block of mass M is connected to a spring of spring constant k. The system is placed on a frictionless horizontal surface and the other end of the spring is firmly fixed to a wall so the block, when displaced from equilibrium a distance A and then released from rest, will experience simple harmonic motion. A second block of mass m is then placed on top of the first block. The coefficient of static friction associated with the interaction between the two blocks is μ_S . What is the maximum value A can be so the blocks oscillate together without the top block slipping on the bottom block?

17. A block of mass $M = 0.800\,\text{kg}$ is connected to a spring of spring constant $k = 2.00\,\text{N/m}$. The system is placed on a frictionless horizontal surface and the other end of the spring is firmly fixed to a wall so the block, when displaced from equilibrium a distance A and then released from rest, will experience simple harmonic motion. A second block of mass $m = 0.600\,\text{kg}$ is then placed on top of the first block. The coefficient of static friction associated with the interaction between the two blocks is $\mu_S = 0.500$. Use $g = 9.80$ m/s^2. (a) What is the maximum value A can be so the blocks oscillate together without the top block slipping on the bottom block? (b) What is the angular frequency in this situation?

Exercises 18 – 23 deal with various aspects of the situation shown in Figure 12.21.

18. A block with a mass of 0.500 kg is released from rest from the top of a ramp that has the form of a 3-4-5 triangle, measuring 3.00 m high and having a base of 4.00 m, as shown in Figure 12.21. The block then slides down the incline, encountering a spring with a spring constant of 5.00 N/m after sliding for 2.50 m. Neglect friction and use g = 9.80 m/s^2. Where does the block reach its maximum speed, at the point it first makes contact with the spring, or at a point higher up the ramp or lower down the ramp than where it first makes contact with the spring? Briefly justify your answer.

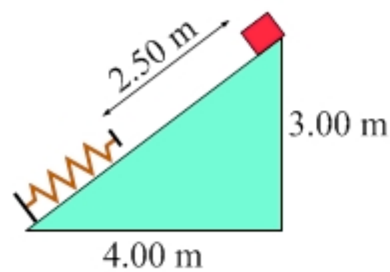

Figure 12.21: A block released from rest from the top of a ramp slides for a distance of 2.50 m before it encounters a spring. For Exercises 18 – 23.

19. Return to the situation described in Exercise 18. Determine (a) how far the block has slid down the ramp when the block reaches its highest speed; (b) the value of this maximum speed.

20. Return to the situation described in Exercise 18. Find the maximum compression of the spring in this situation.

21. Return to the situation described in Exercise 18, but now we'll add friction between the block and the ramp. The coefficient of kinetic friction between the block and the ramp is 0.400. Where does the block reach its maximum speed now, at exactly the same point on the ramp it did in Exercise 18, or at some place higher up or lower down than this point? Briefly justify your answer.

22. Return to the situation described in Exercise 18, but now we'll add friction between the block and the ramp. The coefficient of kinetic friction between the block and the ramp is 0.400. Determine (a) how far the block has slid down the ramp when the block reaches its highest speed; (b) the value of this maximum speed.

23. Return to the situation described in Exercise 18, but now we'll add friction between the block and the ramp. The coefficient of kinetic friction between the block and the ramp is 0.400. Find the maximum compression of the spring in this situation.

Exercises 24 – 30 deal with energy and energy conservation in oscillating systems.

24. A block with a mass of 0.500 kg that is attached to a spring is oscillating back and forth on a frictionless horizontal surface. The period of the oscillations is 2.00 s. When the block is 30.0 cm from its equilibrium position, its speed is 1.20 m/s. What is the amplitude of the oscillations?

25. Consider again the system described in Exercise 24. At a time of $T/4$ after being released from rest, the block is passing through the equilibrium position. At a time of $T/8$ after being released, determine the system's (a) elastic potential energy; (b) kinetic energy.

26. Consider again the system described in Exercise 24, but now we'll make it more realistic. There is a small coefficient of friction associated with the interaction between the block and the surface. This means that, over time, the amplitude of the oscillations decrease until eventually the block comes to rest and remains at rest. Approximately how much work is done by friction on the block during this process?

27. A block is attached to a spring, displaced from equilibrium a distance of 0.800 m, and released from rest. It then oscillates on a frictionless horizontal surface with a period of 4.00 s. At the instant the block is released from rest, the energy in the system is all elastic potential energy, as shown in the set of energy bar graphs in Figure 12.22(a). (a) At how many locations in the subsequent oscillations do we get the set of energy bar graphs shown in Figure 12.22(b)? (b) Determine the distance of each of these locations from the equilibrium position.

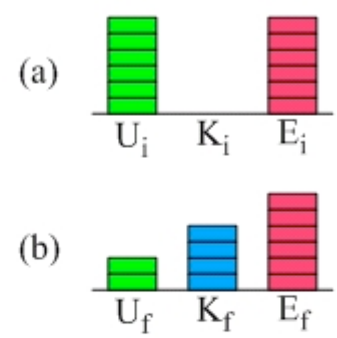

Figure 12.22: Energy bar graphs for a block on a spring at (a) its release point, and (b) some other point. For Exercises 27 and 28.

28. Return to the situation described in Exercise 27. (a) During one complete oscillation, at how many different times does the energy correspond to the set of energy bar graphs shown in Figure 12.22(b)? (b) Assuming the block is released from rest at $t = 0$, determine all the times during the first complete oscillation when the system's energy corresponds to the energy bar graphs shown in Figure 12.22(b).

29. A ball is tied to a string to form a simple pendulum with a length of 1.20 m. The ball is displaced from equilibrium by some angle θ_i and released from rest. In the subsequent oscillations, the ball's maximum speed is 2.50 m/s. (a) From what height above equilibrium was the ball released? (b) What is θ_i ? Use $g = 9.80$ m/s^2.

30. As shown in Figure 12.23 a simple pendulum with a length of 1.00 m is released from rest from an angle of 20° measured from the vertical. When the ball passes through its equilibrium position the string hits a peg, effectively shortening the length of the pendulum to 50.0 cm. (a) How does the maximum height reached by the pendulum on the right compare to the height of the ball at its release point on the left? Justify your answer. (b) What is the maximum angle, measured from the vertical, of the string when the ball is on the right?

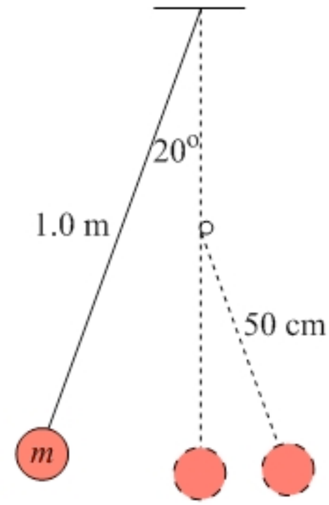

Figure 12.23: A simple pendulum swings down through its equilibrium position, and then hits a peg that effectively shortens its length to 50 cm. For Exercises 30 and 31.

Exercises 31 – 35 deal with time and simple harmonic motion.

31. Return to the situation described in Exercise 30 and shown in Figure 12.23. What is the period of one complete oscillation for this pendulum? Use $g = 9.80$ m/s^2.

32. A block with a mass of 0.600 kg is connected to a spring, displaced in the positive direction a distance of 50.0 cm from equilibrium, and released from rest at $t = 0$. The block then oscillates without friction on a horizontal surface. The first time the block is a distance of 15.0 cm from equilibrium is at $t = 0.200$ s. Determine (a) the period of oscillation; (b) the value of the spring constant; (c) the block's velocity at $t = 0.200$ s; and (d) the block's acceleration at $t = 0.200$ s.

33. Repeat Exercise 32, but now $t = 0.200$ s represents the second time the block is a distance of 15.0 cm from equilibrium.

34. A block on a spring is released from rest from a distance A from equilibrium at $t = 0$. The block then experiences simple harmonic motion with a period T. Determine all the times during the first complete oscillation when the block is a distance $A/4$ from equilibrium.

35. A block with a mass of 0.800 kg is connected to a spring, displaced in the positive direction a distance of 40.0 cm from equilibrium, and released from rest at $t = 0$. The block then oscillates without friction on a horizontal surface. At a time of $t = 0.500$ s, the block is 30.0 cm from equilibrium. If the block's period of oscillation is longer than 0.500 s, determine the spring constant of the spring. Find all possible answers.

Exercises 36 – 39 combine collisions with simple harmonic motion situations.

36. A wheeled cart with a mass of 0.50 kg is rolling along a horizontal track at a constant velocity of 2.0 m/s when it experiences an elastic collision with a second identical cart that is initially at rest, and attached to a spring with a spring constant of 4.0 N/m. This situation is illustrated in Figure 12.24. After the collision, the second cart moves to the right. (a) What is the first cart doing after the collision? Briefly justify your answer. (b) What is the maximum compression of the spring? (c) There is a second collision between the carts. What is each cart doing after the second collision?

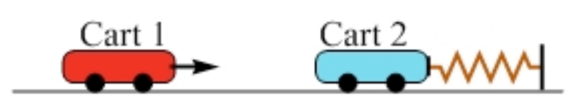

Figure 12.24: A cart collides with a second cart that is initially at rest and attached to a spring. For Exercises 36 and 37.

37. A wheeled cart with a mass of 0.50 kg is rolling along a horizontal track at a constant velocity of 2.0 m/s when it experiences a collision with a second identical cart that is initially at rest, and attached to a spring with a spring constant of 4.0 N/m. This situation is illustrated in Figure 12.24. After the collision, the carts stick together and move as one unit. What is the maximum compression of the spring?

38. In a spring version of the ballistic pendulum situation we looked at in Chapter 7, a wooden block with a mass of 0.500 kg is attached to a spring with a spring constant of $k = 600$ N/m. As shown in Figure 12.25, the system is placed on a frictionless horizontal surface with the block at rest at the equilibrium position. A bullet with a mass of 30.0 g is fired at the block. The bullet gets embedded in the block and, after the collision, the block experiences simple harmonic motion with an amplitude of 15.0 cm. Assuming the bullet's velocity is horizontal at the instant the collision takes place, what is the speed of the bullet just before it hits the block?

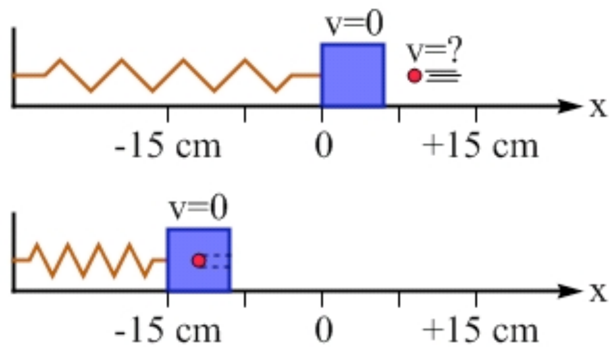

Figure 12.25: A bullet embeds itself in a block. The subsequent oscillations take place with an amplitude of 15.0 cm. For Exercise 38.

39. As shown in Figure 12.26, a ball of mass m is tied to a string to form a simple pendulum. The ball is displaced from equilibrium so the angle between the string and the vertical is 60˚, and is then released from rest. It swings down, and at its lowest point it collides with a second ball of mass $4m$ that is initially at rest on the edge of a table. If the collision is elastic and the second ball strikes the floor at a point 1.50 m vertically lower and 1.20 m horizontally from where it started, find the length of the string the first ball is attached to.

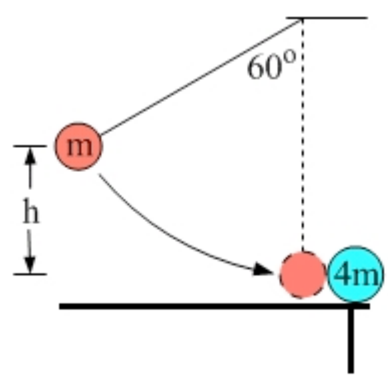

Figure 12.26: A ball of mass m, tied to a string to form a simple pendulum, swings down and collides elastically with a ball of mass $4m$. For Exercise 39.

General Problems and Conceptual Questions

40. A block of mass m is connected to a spring of spring constant k, displaced a distance A from equilibrium and released from rest. An identical block is connected to a spring of spring constant $4k$ and released from rest so its total mechanical energy is equal to that of the first block-spring system. (a) Assuming the blocks are simultaneously released from rest and that they each experience simple harmonic motion horizontally, sketch graphs of the total mechanical energy, elastic potential energy, and kinetic energy as a function of displacement from equilibrium. (b) Repeat part (a), but this time sketch graphs of the three types of energy as a function of time instead.

41. Return to the situation described in Exercise 40. Now plot graphs, as a function of time, of the (a) position; (b) velocity; and (c) acceleration for both of the blocks.

42. You are trying to demonstrate to your friend the connection between uniform circular motion and simple harmonic motion. You have a motorized turntable that spins at a rate of exactly 1 revolution per second, and you glue a ball to the turntable at a distance of 50 cm from the center. You then take a small bucket with a mass of 100 g and connect it to a spring that has a spring constant of 4.00 N/m. The bucket will then oscillate back and forth on a frictionless surface. (a) What mass of sand should you place in the bucket so the period of oscillation of the bucket matches the period of revolution of the ball on the turntable? (b) Assuming you have lined up the equilibrium position of the bucket on the spring with the center of the turntable, what amplitude should you give the bucket so its motion exactly matches one component of the motion of the ball on the turntable?

43. You match the motion of two objects, a ball glued to a turntable that is rotating at a constant angular velocity and a block oscillating on a frictionless surface because it is connected to a spring. (a) If the period of the block's oscillations is 2.5 s, what is the angular frequency of the turntable? (b) If you replace the spring by a spring with double the original spring constant, what should the angular frequency of the turntable be?

44. Consider the two graphs shown in Figure 12.27 for a block that oscillates back and forth on a frictionless surface because it is connected to a spring. The graph on the left shows the block's displacement from equilibrium, as a function of time, for one complete oscillation of the block. The graph on the right shows the elastic potential energy stored in the spring, as a function of time, over the same time period. (a) What is the spring constant of the spring? (b) What is the mass of the block? (c) What is the maximum speed reached by the block as it oscillates?

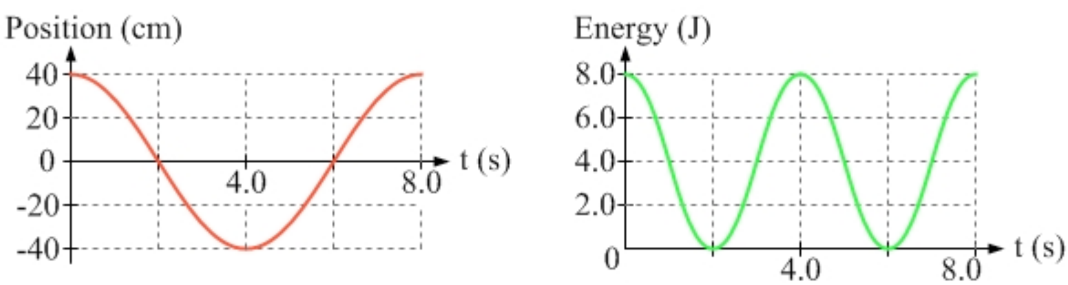

Figure 12.27: Graphs of position as a function of time and elastic potential energy as a function of time for a block oscillating horizontally on a spring. For Exercises 44 and 45.

45. Using only the information available to you in the position vs. time graph shown in Figure 12.27, determine (a) the maximum speed of the oscillating block; (b) the magnitude of the maximum acceleration of the oscillating block.

46. A block with a mass of 500 g is connected to a spring with a spring constant of 2.00 N/m. You start the motion by hitting the block with a stick so that, at $t = 0$, the block is at the equilibrium position but has an initial velocity of 2.00 m/s in the positive direction. The block then oscillates back and forth without friction. Over two complete cycles of the resulting oscillation, plot, as a function of time, the block's (a) position; (b) velocity; and (c) acceleration.

47. Equations 12.3 – 12.5 are ideal for describing the motion of an object experiencing simple harmonic motion after having been displaced from equilibrium and released from rest at $t = 0$. Modify the three equations so they match the motion described in Exercise 46.

48. As shown in Figure 12.28, two springs are separated by a distance of 1.60 m when the springs are at their equilibrium lengths. The spring on the left has a spring constant of 15.0 N/m, while the spring on the right has a spring constant of 7.50 N/m. A block, with a mass of 400 g, is then placed against the spring on the right, compressing it by a distance of 20.0 cm, which also happens to be the width of the block. The block is then released from rest. (a) Assuming all the energy initially stored in the spring is transferred to the block, and that the horizontal surface is frictionless, how long will it take until the block returns to its release point? (b) Repeat the question, but now assume that the block is held against the spring on the left, and released from rest after compressing that spring by 20 cm.

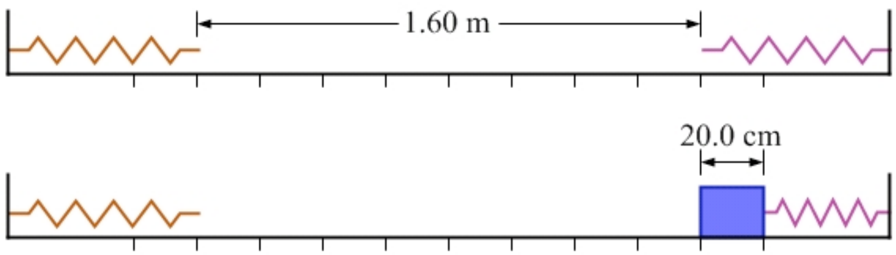

Figure 12.28: Two springs are separated by a distance of 1.60 m. A block is then held against one spring and released from rest. For Exercise 48.

49. A ballistic cart is a cart containing a ball on a compressed spring. In a popular demonstration, the cart is rolled with a constant horizontal velocity past a trigger, which causes the spring to be released, firing the ball vertically (with respect to the cart) into the air. The ball then lands in the cart again 0.60 s later. If the ball has a mass of 22 grams and the spring is initially compressed by 7.5 cm, determine the spring constant of the spring. Assume all the energy stored in the spring is transferred to the ball, and use $g = 9.8$ m/s^2.

50. A block with a mass of 0.600 kg is connected to a spring with a spring constant of 4.50 N/m. The block is displaced a distance A from equilibrium and released from rest. How long after being released is the block first (a) at the equilibrium position? (b) at a point $A/4$ from equilibrium?

51. A block on a spring of spring constant $k = 12.0$ N/m experiences simple harmonic motion with a period of 1.50 s. What is the block's mass?

52. Consider again the situation described in Exercise 51. Such a system is used by astronauts in orbit to measure their own masses. Do a web search for "body mass measurement device" (the name of this system) and write a paragraph or two describing how it works.

53. Among the many things that Galileo Galilei is known for are his observations about pendula. Do some research about Galileo and write a paragraph or two describing his contributions to our understanding of the simple pendulum.

54. A particular wooden block floats in water with 30% of its volume submerged. You then push the block farther under the water so that 40% of its volume is submerged. When you let go the block bobs up and down. (a) For simple harmonic motion, there must be a restoring force proportional, and opposite in direction, to the displacement from equilibrium. Considering the net force on the block from combining the buoyant force and the force of gravity, does that net force fit the requirement necessary for simple harmonic motion? (b) Write an expression for the angular frequency of the block's oscillations.

55. Return again to the situation described in Exercise 54. The block has a mass of 0.30 kg. (a) Draw a free-body diagram showing the forces acting on the block immediately after it is released from rest. (b) Using $g = 10$ m/s^2, determine the block's initial acceleration. (c) Describe what happens to the block's free-body diagram as the block moves.

56. You are at the playground with a young boy who has a mass of 20 kg. When the boy is on a swing, you observe that you push him exactly once every 2.0 s. How long are the ropes attaching the swing to its support? Use $g = 9.8$ m/s^2.

57. A simple pendulum consists of a ball with a mass of 0.500 kg attached to a string of length L. The ball is displaced from equilibrium so that, when the ball is released from rest, it is at a level 1.00 m above its equilibrium position, and the string makes a 60˚ angle with the vertical. Use $g = 9.80$ m/s^2. (a) What is the length of the string? (b) Apply energy conservation to find the speed of the ball as it passes through the equilibrium position. (c) Using the small-angle approximation, it can be shown that the maximum speed of the pendulum ball is given by $v_{max} = L\theta_{max}\omega$. Making sure that your units are correct, use this equation to check your answer to part (b). (d) Your results in parts (b) and (c) should be close but should not agree exactly. Comment on which answer is better and why there is any disagreement at all.

58. Return to the situation described in Exercise 57. What is the tension in the string when the ball passes through the equilibrium position?

59. Return to the situation described in Exercise 57. After many oscillations, air resistance and friction eventually bring the pendulum to a stop. What is the total work done by resistive forces in this situation?

60. As shown in Figure 12.29, two simple pendula are identical except that the mass of the ball on one pendulum is 3 times the mass of the ball on the other. Each pendulum has a length of 1.5 m. The pendula are displaced by angles of 20˚, but in opposite directions, and simultaneously released from rest. The balls then experience an elastic collision with one another. (a) What is the velocity of each ball immediately after the first collision? (b) The balls experience a second elastic collision. What is the maximum angular displacement reached by each ball as a direct result of this second collision? (c) Describe, in general, how the motion proceeds after that.

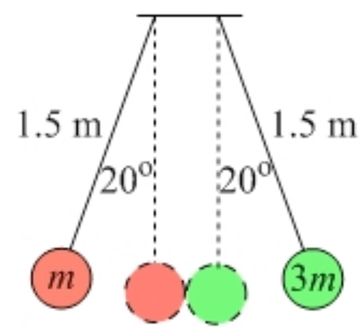

Figure 12.29: Two simple pendula, one with a ball of mass m and the other with a ball of mass $3m$, are displaced by 20˚ and released from rest. They swing down and collide with one another. What happens next? For Exercises 60 and 61.

61. Consider again the situation described in Exercise 60. Determine the time taken by each phase of the motion.

62. A grandfather clock uses a pendulum to keep time. For this exercise, treat the pendulum as a simple pendulum and use $g = 9.80$ m/s^2. (a) How long should the pendulum be if its period needs to be exactly 4 seconds for the clock to keep accurate time? (b) You now have two identical grandfather clocks, both set to keep accurate time on Earth. You take one to the Moon, where the magnitude of the gravitational field is 1/6 what it is on Earth. The clocks are started simultaneously when they both read 12 o'clock. One hour later, the clock on the Earth reads 1 o'clock. What is the time shown on the clock on the Moon at that instant?