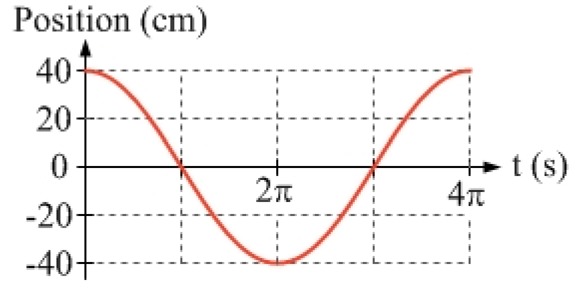
PROBLEM 1 – 15 points

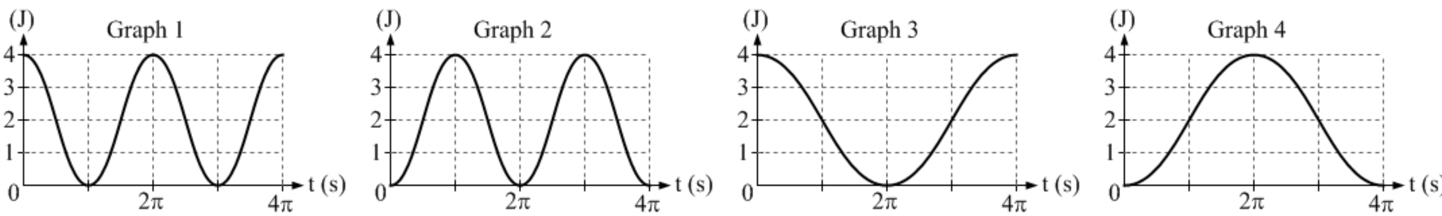
A block attached to a horizontal spring is displaced from the spring's equilibrium position and released from rest. The graph at right shows the block's position as a function of time for one complete oscillation as it oscillates on a frictionless surface.



[3 points] (a) Find ω, the angular frequency, for this system.

The period is $T = 4\pi$ s. $\omega = \frac{2\pi}{T} = \frac{2\pi}{4\pi\ \text{s}} = 0.5$ rad/s

Now consider these graphs.



[2 points] (b) Which graph represents the graph of the elastic potential energy as a function of time for this system?

[**X**] Graph 1 [ ] Graph 2 [ ] Graph 3 [ ] Graph 4

Starts off with maximum potential energy. Hits maximum whenever the block's displacement is maximized, in either direction, and is zero whenever the block passes through $x = 0$.

[2 points] (c) Which graph represents the graph of the block's kinetic energy as a function of time for this system?

[ ] Graph 1 [**X**] Graph 2 [ ] Graph 3 [ ] Graph 4

For one thing, the sum of the elastic potential energy and the kinetic energy equals the total energy, which must be constant because of energy conservation.

Now, the **mass** of the block is **increased by a factor of 4**, and the block is released from rest from the same position as the original block.

[2 points] (d) Compared to the original system, the amplitude of the new oscillations will be…

[ ] larger [**X**] unchanged [ ] smaller

The amplitude depends on where the block is released from, which is the same for both.

[2 points] (e) Compared to the original system, the new system's angular frequency will be…

[ ] larger [ ] unchanged [**X**] smaller

The angular frequency decreases as the mass of the object increases.

[2 points] (f) Compared to the original block, the new block's maximum kinetic energy will be…

[ ] larger [**X**] unchanged [ ] smaller

The maximum kinetic energy will be equal to the initial mechanical energy in the system when the block is released. This is all elastic potential energy, which is the same for both because the spring constant and amplitude are the same.

PROBLEM 2 – 10 points

You have two identical springs and two identical blocks. You attach each block to a spring so you have two spring-block systems, and you set the blocks up to oscillate simultaneously on a frictionless horizontal surface. You pull the blocks so they stretch their respective springs, releasing them both from rest simultaneously. However, when you release the blocks one of them (we'll call this system 1) is displaced a distance *A* from equilibrium and the other (we'll call this system 2) is displaced 2*A* from equilibrium.

[2 points] (a) If the block in system 1 reaches a maximum speed *v* in its oscillations, what is the maximum speed reached by the block in system 2?

[ ] $\frac{v}{2}$ [ ] v [ ] $\sqrt{2}\,v$ [**X**] $2v$ [ ] $4v$

Using energy conservation, $\frac{1}{2}kA^2 = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{k}{m}}A$**, so doubling the amplitude doubles the maximum speed.**

[2 points] (b) If the block in system 1 experiences oscillations with a period T, what is the period of the oscillations experienced by the block in system 2?

[ ] $\frac{T}{2}$ [**X**] T [ ] $\sqrt{2}\,T$ [ ] $2T$ [ ] $4T$

The angular frequency, and the period, are independent of amplitude.

[2 points] (c) If the maximum force experienced by the block in system 1 is $F_{\max}$, what is the maximum force experienced by the block in system 2?

[ ] $\frac{F_{\max}}{2}$ [ ] $F_{\max}$ [ ] $\sqrt{2}\,F_{\max}$ [**X**] $2\,F_{\max}$ [ ] $4\,F_{\max}$

The maximum force has a magnitude of kA, so doubling A will double the maximum force.

[2 points] (d) If the potential energy stored in the spring in system 1 is U_i when the block is first released from rest, what is the potential energy initially stored in the spring in system 2?

[ ] $\frac{U_i}{2}$ [ ] U_i [ ] $\sqrt{2}\,U_i$ [ ] $2U_i$ [**X**] $4U_i$

$$U_i = \frac{1}{2}kA^2 \quad so \quad U_{i2} = \frac{1}{2}k(2A)^2 = 4\times\frac{1}{2}kA^2 = 4U_i$$

[2 points] (e) At a particular instant, some time after being released, the block in system 1 is 20 cm from its equilibrium position. How far from equilibrium is the block in system 2 at that same instant?

[ ]10 cm [ ] 20 cm [**X**] 40 cm

[ ] there is not enough information to answer this question

With the blocks having the same period, and being released from rest at the same time, the block in system 2 will always be twice as far from equilibrium as the block in system 1 is.

$x_1 = A\cos(\omega t)$ and $x_2 = 2A\cos(\omega t)$

PROBLEM 3 – 15 points

You have a mass on a vertical spring, and a simple pendulum that undergoes small-amplitude oscillations. At this location on Earth they both oscillate with a period of exactly 1.00 seconds. Now you take them to Planet Zorg where the value of *g* is exactly 4 times smaller than *g* at this location on Earth.

[3 points] (a) What is the period when the mass on the vertical spring oscillates on Zorg?

The angular frequency, and the period, depend on the mass and on the spring constant – there is no dependence on g, the acceleration due to gravity. This, the period is still 1.00 s.

[3 points] (b) What is the period of small-amplitude oscillations for the simple pendulum on Zorg?

The equation for the period of a pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$**, so if g increases by a factor of 4, the period decreases by a factor of 2. The period on Zorg is 0.500 s.**

[4 points] (c) If you give the pendulum the same angular displacement from equilibrium on Earth and on Zorg and release it from rest in both cases, how will the speed of the pendulum bob at its lowest point compare on the two planets?

We can do this one by energy conservation. The pendulum bob is falling through the same height on both planets, so we have:

$$mgh = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{2gh}$$

If *g* increases by a factor of 4, *v* increases by a factor of 2. Thus, the speed at the lowest point in the motion on Zorg is twice that at the lowest point of the motion on Earth.

[5 points] (d) The simple pendulum can be used in various ways to determine the local value of *g*. Can the mass on the vertical spring be used to measure *g*? If so, describe one way to do it.

Yes. One way is to hang the mass vertically from the spring, and measure how far the spring is stretched when the mass is at equilibrium. At equilibrium, the free-body diagram of the mass has *mg* directed down and *kx* directed up, and these forces balance one another, Setting these two forces equal allows us to solve for *g*:

$$mg = kx \quad \Rightarrow \quad g = \frac{kx}{m}$$