Answer to Essential Question 11.8: Review the analysis in step 3 of Exploration 11.8. Both the mass and radius cancel out of the energy conservation equation. This tells us, surprisingly, that the mass and radius are irrelevant. In other words, all uniform solid spheres roll the same, all uniform solid disks (or cylinders) roll the same, and all rings roll the same – all the races involving two of the same kind of object end in a tie.

11-9 Rotational Impulse and Rotational Work

Let's continue our method of determining rotational equations from their straight-line motion counterparts by writing down expressions for rotational impulse and rotational work. In Chapter 6, the impulse relationship we came up with was: $\Delta\vec{p} = \vec{F}_{net}\,\Delta t$. In words, this equation tells us that the change in momentum an object experiences is equal to the product of the net force applied to the object multiplied by the time interval over which it is applied. Transforming this to a rotational setting, an object's change in angular momentum is equal to the net torque it experiences multiplied by the time interval over which that net torque is applied:

$\Delta\vec{L} = \vec{\tau}_{net}\,\Delta t$. (Equation 11.4: **Rotational impulse**)

Similarly, we can consider the concept of work in a rotational setting. For straight-line motion, if we meld the work equation with the work-energy theorem we get:

$\Delta K = W_{net} = \vec{F}_{net} \bullet \Delta\vec{r} = F_{net}\,\Delta r \cos\phi$. (Equation 6.8: **Work-kinetic energy theorem**)

In chapter 6, we used the variable θ to represent the angle between the net force $\vec{F}_{net}$ and the displacement $\Delta\vec{r}$. We'll use ϕ here instead because in this chapter we're using θ to represent the angular position of a rotating object.

To find the expression for work in a rotational setting, start with equation 6.8. Replace force $\vec{F}$ by its rotational equivalent, $\vec{\tau}$, and replace displacement $\Delta\vec{r}$ by its rotational equivalent $\Delta\vec{\theta}$. This gives:

$\Delta K = W_{net} = \vec{\tau}_{net} \bullet \Delta\vec{\theta} = \tau_{net}\,\Delta\theta\,\cos\phi$. (Equation 11.5: **Rotational work**)

If the dot product notation confuses you, feel free to ignore it! Because we'll deal only with rotation about one axis (rotation in one dimension), we can make Equation 11.5 simpler:

$\Delta K = W_{net} = \pm\tau_{net}\,\Delta\theta$. (Eq. 11.6: **Work-kinetic energy theorem, for rotation**)

We use the plus sign when the torque is in the same direction as the angular displacement, and the minus sign when the torque is opposite to the direction of the angular displacement.

EXAMPLE 11.9 – Comparing the motions

Note – compare this example to Example 6.3. The methods of analysis in that example and this one are virtually identical. Two objects, *A* and *B*, are initially at rest. The objects have the same mass and radius. Object *A* is a uniform solid disk, while object *B* is a bicycle wheel that can, for this purpose, be considered to be a ring. Each object rotates with no friction about an axis through its center, perpendicular to the plane of the disk/wheel. Identical net torques are then applied to the objects by pulling on strings wrapped around their outer rims. Each net torque is removed once the object it is applied to has accelerated through one complete rotation.

(a) After the net torques are removed which object has more kinetic energy?
(b) After the net torques are removed which object has more speed?
(c) After the net torques are removed, which object has more momentum?

SOLUTION

(a) A diagram of this situation is shown in Figure 11.20. Because the objects start from rest, the angular displacement of each is in the same direction as the net torque (clockwise, in the case shown in Figure 11.20). Because the objects experience equal torques and equal angular displacements the work done on the objects is the same, by Equation 11.6. This means the change in kinetic energy is the same for each, and, because they both start with no kinetic energy, their final kinetic energies are equal.

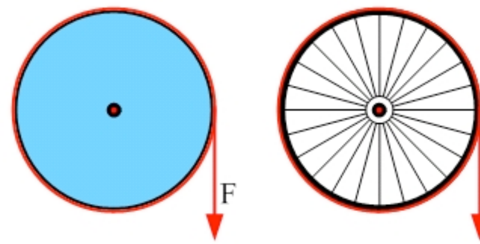

Figure 11.20: Diagrams of the disk and wheel. Each object starts from rest and rotates about an axis perpendicular to the page passing through the center of the object. The force exerted on the string wrapped around the object is removed once the object has accelerated through exactly one revolution.

(b) Unlike Example 6.3, in which the objects had different masses, these objects have the same mass M and the same radius R. This is a rotational situation, however, so what matters is how their rotational inertias compare. Object A, a uniform solid disk rotating about an axis through its center, has a rotational inertia of $I_A = \frac{1}{2}MR^2$. Object B, which we are treating as a ring, has a rotational inertia of $I_B = MR^2$. Thus the relationship between the rotational inertias is $I_A = \frac{1}{2}I_B$. If the objects have the same kinetic energy but B has a larger rotational inertia then A must have a larger angular speed. Setting the final kinetic energies equal, $K_A = K_B$, gives:

$$\frac{1}{2}I_A\omega_A^2 = \frac{1}{2}I_B\omega_B^2 .$$

Canceling factors of $\frac{1}{2}$ gives: $I_A\omega_A^2 = I_B\omega_B^2$

Bringing in the relationship between the rotational inertias gives: $\frac{1}{2}I_B\omega_A^2 = I_B\omega_B^2$.

This gives $\omega_A = \sqrt{2}\,\omega_B$, so object A has a larger angular speed than object B.

(c) One way to find the angular momenta is as follows:

$$\vec{L}_A = I_A\vec{\omega}_A = \frac{1}{2}I_B\vec{\omega}_A = \frac{1}{2}I_B\left(\sqrt{2}\,\vec{\omega}_B\right) = \frac{1}{\sqrt{2}}I_B\vec{\omega}_B = \frac{1}{\sqrt{2}}\vec{L}_B .$$

Thus, object B, the wheel, has a larger angular momentum than object A, the disk. As in Example 6.3, we can understand this result conceptually. The change in angular momentum is the net torque multiplied by the time over which the net torque acts. Both objects experience identical torques, but because B has a larger rotational inertia, B takes more time to spin through one revolution than A does. Because the torque is applied to B for a longer time, B's change in angular momentum, and final angular momentum, has a larger magnitude than A's.

Related End-of-Chapter Exercises: 22, 23.

Essential Question 11.9: Return to the situation described in Example 11.9, but now object B is replaced by object C, a bicycle wheel of the same mass as object A but with a different radius. Once again, we can treat the bicycle wheel as a ring. The situation described in Example 11.9 is repeated, but this time objects A and C end up with the same rotational kinetic energy and the same angular momentum. How is this possible? Be as quantitative about your answer as you can.