

Answer to Essential Question 11.6: One way for Sarah to jump onto the merry-go-round, without causing the merry-go-round to spin, is for Sarah to direct her velocity at the center of the merry-go-round, instead of tangent to it. If Sarah ran directly toward the center of the merry-go-round she would have no angular momentum before the collision and there would be no reason for the system to spin after the collision.

11-7 Considering Conservation, and Rotational Kinetic Energy

In step 4 of Exploration 11.6, we stated that the angular momentum of the system consisting of Sarah and the merry-go-round was conserved, because no external torques were acting on the system. Let's justify that statement. We do not have to concern ourselves with vertical forces, such as the force of gravity or the normal force applied to the merry-go-round by the ground, because vertical forces give no torque about a vertical axis of rotation. We also do not have to concern ourselves with the force that Sarah exerts on the merry-go-round, or the equal-and-opposite force the merry-go-round exerts on Sarah, because the system we're considering consists of the combination of Sarah and the merry-go-round, so those are internal forces and cancel one another. Still, let's examine those forces a little.

Individual free-body diagrams for Sarah and the merry-go-round when Sarah first jumps on the merry-go-round are shown in Figure 11.17. Through some combination of friction between her shoes and the merry-go-round, and a contact force between her hands and any handholds on the merry-go-round, there is a force component that acts to the left on Sarah from the merry-go-round (this reduces her speed), and an equal-and-opposite force component that acts to the right on the turntable by Sarah (providing the torque that gives the merry-go-round an angular acceleration). However, the turntable does not accelerate to the right. This is because there is a horizontal force applied on the turntable by whatever the turntable's axis is connected to, which we can consider to be the Earth. As shown in Figure 11.17, the Sarah/merry-go-round system has a net external force acting on it at this point, which is why the *linear* momentum of the system is not conserved. However, this net external force gives rise to no torque about an axis through the center of the merry-go-round, because the force passes through that axis. Because there is no net external torque acting on the system, the system's angular momentum is conserved.

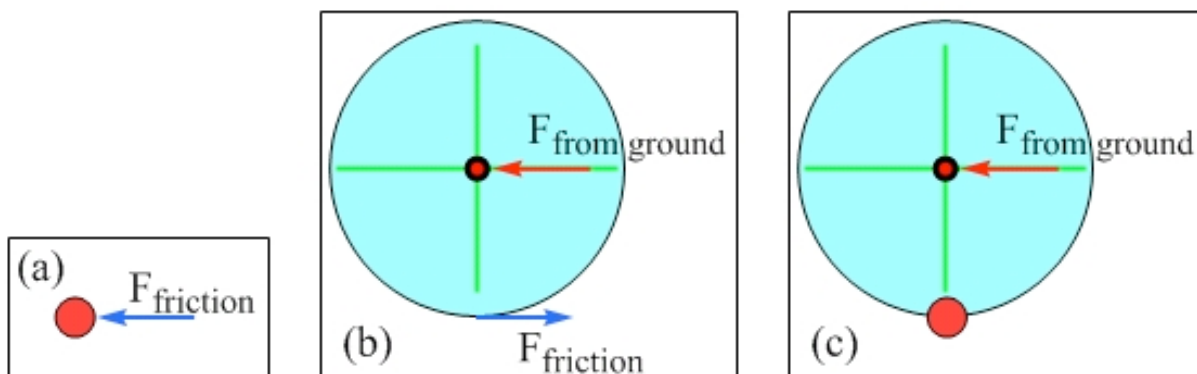


Figure 11.17: Free-body diagrams for Sarah, the merry-go-round, and the system consisting of Sarah and the merry-go-round together, when Sarah initially makes contact with the merry-go-round. Vertical forces are ignored in this overhead view.

Rotational Kinetic Energy

Let's now move from the rotational equivalent of linear momentum to the rotational equivalent of translational kinetic energy. The equation we used previously for kinetic energy is $K = \frac{1}{2}mv^2$. We can find the equivalent expression for kinetic energy in a rotational setting by replacing mass m by rotational inertia I , and speed v by angular speed ω . The kinetic energy of a purely rotating object is thus given by:

$$K = \frac{1}{2}I\omega^2. \quad (\text{Equation 11.3: Rotational kinetic energy})$$

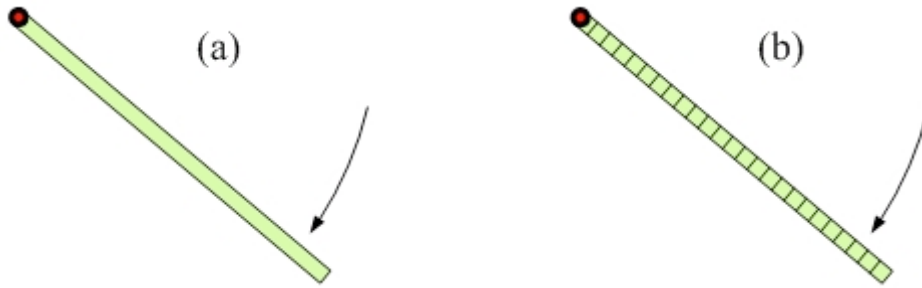


Figure 11.18: (a) A rod that has been released from rest when it was horizontal is now moving. We can find its kinetic energy by breaking the rod into small pieces, as shown in (b), finding the kinetic energy of each piece, and adding these kinetic energies together to find the net kinetic energy.

Let's make sure our substituting-the-equivalent-rotational-variables method of arriving at rotational equations makes sense. Consider, for instance, a uniform rod that can rotate about an axis through one end. If we hold the rod horizontal and then release it from rest, the rod swings down. What is the rod's kinetic energy at a particular instant, say at the instant shown in Figure 11.18 (a)? One thing we could do is, as shown in Figure 11.18 (b), break the rod into small pieces of mass m_i , determine the speed v_i of each piece, find the kinetic energy $\frac{1}{2}m_i v_i^2$ of each piece, and then add up all these kinetic energies to find the total kinetic energy:

$$K = \sum \frac{1}{2}m_i v_i^2.$$

Because the speed of each piece is different, while the angular speed of each piece is the same, let's write the sum in terms of the rod's angular speed instead:

$$K = \sum \frac{1}{2}m_i (r_i \omega)^2.$$

If we bring the constants of $\frac{1}{2}$ and ω^2 out in front of the sum, our expression becomes $K = \frac{1}{2}\omega^2 \sum m_i r_i^2$, which we can write as $K = \frac{1}{2}I\omega^2$, because the definition of rotational inertia is $I = \sum m_i r_i^2$. This expression for the kinetic energy agrees with what we came up with above (and it works for any rotating object, not just a rod!).

Essential Question 11.7: In Chapter 7 we used names such as “elastic collision” and “inelastic collision” to classify various collisions. Under what category would the Sarah/merry-go-round collision described in the previous Exploration fall?