Answer to Essential Question 11.5: No friction force can act on the ball, so the correct free-body diagram is that shown in Figure 11.14 (c). A force of friction in the direction of motion would increase the ball's translational speed, and the counterclockwise torque from the force of friction would decrease the ball's angular speed. A force of static friction directed opposite to the ball's velocity would decrease the translational speed while increasing the rotational speed. The ball rolls horizontally at constant velocity only if no friction force acts.

11-6 Angular Momentum

By now, we have looked at enough analogies between straight-line motion and rotational motion that we can simply take a straight-line motion equation, replace the straight-line motion variables by their rotational counterparts, and write down the equivalent rotational equation. We could also derive the rotational equations following a derivation parallel to the one we used for the straight-line motion equation, but the end result would be the same.

Let's try this for angular momentum. In Chapter 6, we used the following expression for the linear momentum, $\vec{p}$, of an object of mass *m* moving with velocity $\vec{v}$: $\vec{p} = m\vec{v}$.

Using the symbol $\vec{L}$ to represent angular momentum, we can come up with the equivalent expression for angular momentum by replacing mass *m* by its rotational equivalent, rotational inertia *I*, and velocity $\vec{v}$ by its rotational equivalent $\vec{\omega}$:

$$\vec{L} = I\vec{\omega} . \qquad \text{(Equation 11.1: \textbf{Angular momentum})}$$

We made a number of statements about momentum in Chapter 6. Equivalent statements apply to angular momentum, including:

- Angular momentum is a vector, pointing in the direction of angular velocity.
- The angular momentum of a system can be changed by applying a net torque.
- If no net torque acts on a system, its angular momentum is conserved.

Let's explore this idea of angular momentum conservation.

EXPLORATION 11.6 – Jumping on the merry-go-round

A little red-haired girl named Sarah, with mass *m*, runs toward a playground merry-go-round, which is initially at rest, and jumps on at its edge. Sarah's velocity $\vec{v}$ is tangent to the circular merry-go-round. Sarah and the merry-go-round then spin together with a constant angular velocity $\vec{\omega}_f$. The merry-go-round has a mass *M*, a radius *R*, and has the form of a uniform solid disk. Assume that Sarah's "radius" is small compared to *R*. The goal of this Exploration is to determine an expression for $\vec{\omega}_f$. We can treat this as a collision.

Step 1 – ***Sketch two diagrams, one showing Sarah running toward the merry-go-round and the other showing Sarah and the merry-go-round rotating together after Sarah has jumped on. Imagine that you're looking down on the situation from above.*** These two diagrams are shown in Figure 11.15.

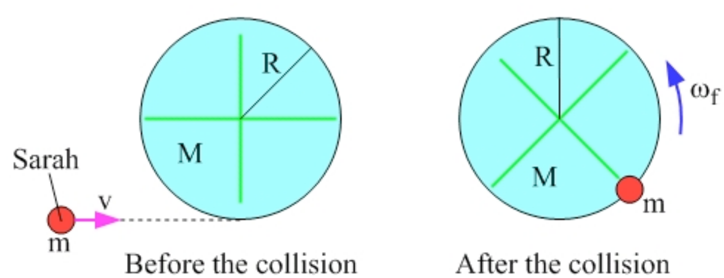

Figure 11.15: On the left is the situation before the collision, as Sarah runs toward the merry-go-round, while on the right is the situation after the collision, with Sarah and the merry-go-round rotating together with a constant angular velocity.

Step 2 – ***What kind of momentum does the Sarah/merry-go-round system have, if any, before Sarah jumps on the merry-go-round? What about after Sarah jumps on?*** After the collision, when the system is rotating, the system clearly has a non-zero angular momentum. Before the collision, however, it is not obvious that the system has any angular momentum, because nothing is rotating. Sarah certainly has a linear momentum, however, because she has a non-zero velocity.

Step 3 – ***Convert Sarah's linear momentum before the collision to an angular momentum, using a method modeled on the way we convert a force to a torque.*** Although there is no rotation before the collision, we can say that the system has an angular momentum with respect to an axis perpendicular to the page that passes through the center of the merry-go-round. Consider how we get torque from force, where the magnitude of the torque is given by $\tau = rF\sin\phi$. Angular momentum is found from linear momentum in a similar fashion, with its magnitude given by:

$L = r\,p\sin\phi = r(mv)\sin\phi$, (Eq. 11.2: **Connecting angular momentum to linear momentum**)

where ϕ is the angle between the line we measure distance along and the line of the linear momentum.

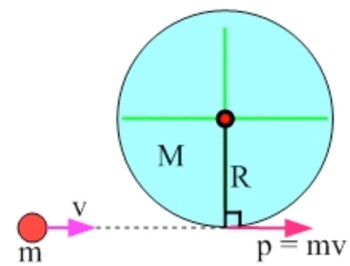

Figure 11.16: The lever-arm method to determine Sarah's angular momentum, with respect to an axis passing through the center of the merry-go-round.

Relative to the axis through the center of the merry-go-round, the angular momentum is: $\vec{L}_i = R\,mv\sin(90°) = R\,mv$, in a counterclockwise direction.

Step 4 – ***Apply angular momentum conservation to express*** $\vec{\omega}_f$ ***, the angular velocity of the system after the collision, in terms of the variables above.*** Angular momentum is conserved because there are no external torques acting on the Sarah/merry-go-round system, relative to a vertical axis passing through the center of the turntable. We will justify this further in section 11-7. Thus, we can say: Angular momentum before the collision = angular momentum afterwards.

The angular momentum afterwards is $\vec{L}_f = I\vec{\omega}_f$. The system's rotational inertia after the collision is the sum of the rotational inertias of Sarah, and the ½ MR^2 of the merry-go-round. Sarah's "radius" is small compared to *R,* so we treat Sarah as a point, assuming that all her mass is the same distance, R , from the center of the turntable. Sarah's rotational inertia is thus mR^2 .

Thus, the rotational inertia of the system after the collision is $I = \frac{1}{2}MR^2 + mR^2$.

Taking counterclockwise to be positive, angular momentum conservation gives: $\vec{L}_i = \vec{L}_f$.

$$+R\,mv = I\vec{\omega}_f = \left(\frac{1}{2}MR^2 + mR^2\right)\vec{\omega}_f .$$

Solving for the final angular velocity of the system gives:

$$\vec{\omega}_f = +\frac{mv}{\frac{1}{2}MR + mR} \quad \text{or,} \quad \vec{\omega}_f = \frac{mv}{\frac{1}{2}MR + mR} \text{ directed counterclockwise.}$$

Key ideas: Linear momentum converts to angular momentum in the same way force converts to torque. Also, we apply momentum conservation ideas to rotational collisions in the same way we analyze collisions in one and two dimensions. **Related End-of-Chapter Exercises: 32, 34, 59.**

Essential Question 11.6: Is it possible for Sarah, with the same initial speed, to jump onto the merry-go-round at the same point, but not make it spin? If so, how could she do this?