Answer to Essential Question 11.3: Many people focus on the counterclockwise torque, relative to an axis perpendicular to the page that passes through the center of the spool, exerted by the force of tension and conclude that the spool rolls to the left. Before jumping to conclusions, however, draw the free-body diagram (after drawing your own, see Figure 11.9). As usual there is a downward force of gravity and an upward normal force. Horizontally there is a force of tension, directed right, exerted by the ribbon. With no friction, the force of tension would cause the spool to move right and spin counterclockwise, so the bottom of the spool would move right with respect to the horizontal surface. Friction must therefore be directed left to oppose this, and, because we know the spool rolls without slipping, the force of friction must be static friction.

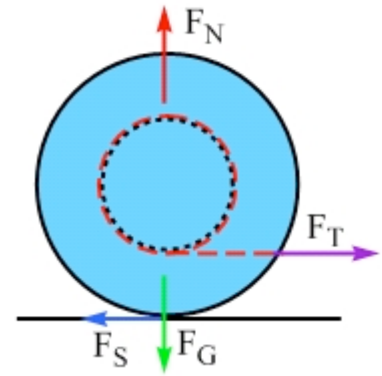


Figure 11.9: The free-body diagram of the spool.

Now we have the complete free-body diagram, we can see that the answer to the question is not obvious. There is one force left and one force right – which is larger? Relative to an axis through the center, there is one torque clockwise and one counterclockwise – which is larger? A quick way to get the answer is to consider an axis perpendicular to the page, passing through the point where the spool makes contact with the horizontal surface. Relative to this axis, three of the four forces give no torque, and the torque from the tension in the string is in a clockwise direction. Clockwise rotation of the spool, relative to the point where the spool touches the surface, is consistent with the spool rolling without slipping to the right. This is opposite to what you would conclude by focusing only on the torque about the center from the force of tension. The spool rolls to the right.

11-4 Combining Rolling and Newton's Second Law for Rotation

Let's now look at how we can combine torque ideas with rolling-without-slipping concepts.

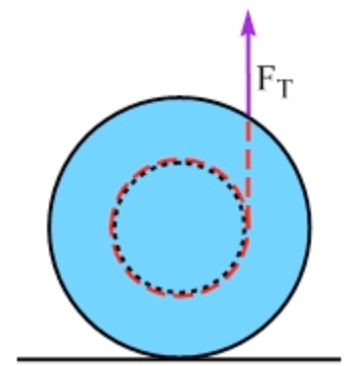


Figure 11.10: An upward force is exerted on the string wrapped around the axle of the spool.

EXPLORATION 11.4 – A vertical force but a horizontal motion

A spool of mass *M* has a string wrapped around its axle. The radius of the axle is half that of the spool. An upward force of magnitude F_T is exerted on the end of the string, as shown in Figure 11.10. This causes the spool, which is initially at rest, to roll without slipping as it accelerates across the level surface.

In which direction does the spool roll? Which horizontal force is responsible for the spool's horizontal acceleration? Let's begin by drawing a free-body diagram of the spool. Figure 11.11 shows a partial free-body diagram, showing only the vertical forces acting on the spool. There is a downward force of gravity acting on the spool, and an upward force of tension applied by the string (note that F_T must be less than or equal to Mg, so the spool has no vertical acceleration). There is also an upward normal force, required to balance the vertical forces.

Is there a horizontal force? If there is, what could it be? Let's go back and think about what is interacting with the spool. The force of gravity accounts for the interaction between the Earth and the spool, and the force of tension accounts for the interaction between the string and the spool. The only interaction left is the interaction between the surface and the spool. The surface exerts a contact force on the spool. Remember that we generally

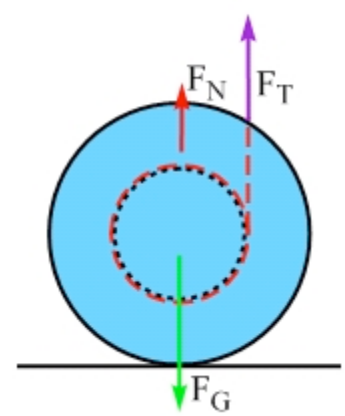


Figure 11.11: A partial free-body diagram of the spool, showing the vertical forces acting on it.

split the contact force into components, the normal force (which we have accounted for) and the force of friction (which we have not).

If there is a horizontal force acting, it can only be a force of friction. Do we need friction in this situation? Consider what would happen if the free-body diagram shown in Figure 11.11 was complete, and there was no friction. Taking an axis perpendicular to the page through the center of the spool, the tension force would give rise to a counterclockwise torque. Because the net force acting on the spool would be zero, however, the spool would simply spin counterclockwise without moving. This is inconsistent with the rolling-without-slipping motion we are told is occurring. There must be a force of friction acting on the spool to cause the horizontal motion.

Note that, without friction, the bottom of the spool rotates to the right relative to the surface. The force of friction must therefore be directed to the left, acting to oppose the relative motion that would occur without friction. Because the force of friction is the only horizontal force acting on the spool, the spool accelerates to the left.

To gain another perspective on this situation, we can follow the procedure discussed in the Answer to Essential Question 11.3, and consider the sum of the torques about the contact point (the point where the spool makes contact with the ground). Both the normal force and the force of gravity pass through the contact point, so they don't give rise to any torques about the contact point. If there is a force of friction, whether it is directed to the right or the left it would also pass through the contact point, giving rise to no torque about that point. Thus, the only force that produces a torque about the contact point is the tension force. Relative to the contact point, this torque is directed counter-clockwise, which is consistent with rolling without slipping to the left. Starting from rest, rolling to the left requires a horizontal force directed to the left, which can only be a friction force.

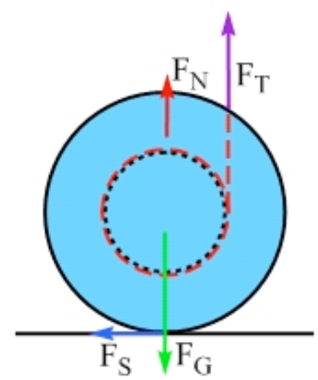
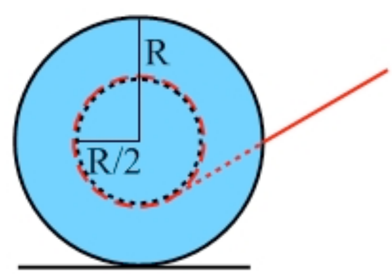


Figure 11.12: The complete free-body diagram of the spool.

Is the force of friction kinetic friction or static friction? Because the spool is rolling without slipping, and the bottom of the spool is instantaneously at rest relative to the surface it is in contact with, the force of friction is the static force of friction. This may sound counter-intuitive, since there is relative motion between the spool as a whole and the surface, but it is very similar to the walking (without slipping) situation that we thought about in Chapter 5. When walking, as long as our shoes do not slip on the floor, a force of static friction acts in the direction of motion. The same thing happens here - in this case, the force of static friction is the only horizontal force acting on the spool, so it is the force accelerating the wheel horizontally. The complete free-body diagram for the rolling-without-slipping situation is shown in Figure 11.12.

Key ideas for rolling without slipping: Rolling without slipping often involves a force of friction, which must be a static force of friction. The static force of friction is often (although not always) in the direction of motion.
Related End-of-Chapter Exercises: 3, 50.



Essential Question 11.4: In the situation shown in Figure 11.13, you pull on the end of a ribbon wrapped around the axle of a spool. Your force is exerted in the direction shown. If the spool rolls without slipping, in which direction does the spool roll?

Figure 11.13: A ribbon is wrapped around the axle of the spool so the ribbon comes off the axle in the direction shown.