Answer to Essential Question 11.2: As we will investigate in more detail in section 11-3, when a wheel rolls without slipping, the point at the bottom of the wheel has the smallest speed (the speed there is zero, in fact), while the point at the top of the wheel is moving fastest.

11-3 Further Investigations of Rolling

Let's continue our analysis of rolling, starting by thinking about the velocity of various points on a wheel that rolls without slipping. We will then go on to investigate rolling spools.

EXPLORATION 11.3 – Determining velocity

Let's turn now from thinking about speeds to thinking about velocities. Consider a wheel rolling without slipping with a constant translational velocity $\vec{v}$, directed to the right, across a level surface. For each point below, determine the point's net velocity by combining, as vectors, the point's translational velocity (the velocity associated with the translational component of the motion) with its velocity because of the rotational component of the motion.

Step 1 – ***Find the net velocity of the center of the wheel.***

Our axis of rotation passes through the center of the wheel. The center of the wheel therefore has no rotational velocity (because $v_{rot} = r\omega$, and $r = 0$). Thus, the net velocity of the center of the wheel is its translational velocity, $\vec{v}$. Note that every point on the wheel has the same translational velocity. Two equal vectors are shown at the center of the wheel in Figure 11.5. One represents the translational velocity at that point, and the other represents the net velocity at that point.

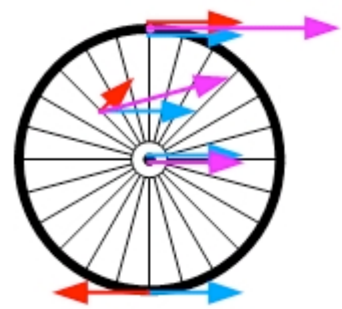

Figure 11.5: The translational (equal vectors all directed right), rotational (tangent to the circle), and net velocities of various points on the wheel. The net velocity at a point is a vector sum of the translational and rotational velocities.

Step 2 – ***Find the net velocity of the point at the very top of the wheel.***

Here, we use the fact that the rotational speed is equal to the translational speed, so we are adding two velocities of equal magnitude. At the top of the wheel, the velocities also point in the same direction, so the net velocity is $2\vec{v}$, as shown in Figure 11.5.

Step 3 – ***Find the net velocity of the point at the very bottom of the wheel.***

At the bottom of the wheel, the rotational velocity exactly cancels the translational velocity, because the vectors point in opposite directions and have equal magnitudes. The net velocity of that point is zero – the point is instantaneously at rest! This is a special condition that is characteristic of rolling without slipping. No slipping implies no relative motion between the surfaces in contact, which means the point at the bottom of the wheel that is in contact with the road surface is at rest.

Figure 11.5 also shows the net velocity at another point on the wheel, a point above and to the left of the center. As with all points, the translational velocity is a vector directed to the right. The velocity associated with the pure rotation is tangent to the circle that passes through the point (and centered at the center of the wheel) - this has a magnitude of $v/2$, because the point is halfway between the center and the rim. The net velocity is the longest of the three vectors at that point, the vector sum of the translational and rotational velocities.

Key ideas for rolling: The net velocity of a point on a rolling wheel can be found by adding, as vectors, the point's translational velocity and its rotational velocity. In the special case of a wheel rolling without slipping with a translational velocity $\vec{v}$, the net velocity of the center of the wheel is $\vec{v}$; while that of the point at the top of the wheel is $2\vec{v}$. A point on the outer edge of the wheel actually comes instantaneously to rest when it reaches the bottom of the wheel.
Related End-of-Chapter Exercises: 5, 6.

EXAMPLE 11.3 – Unrolling a ribbon from a spool

A long ribbon is wrapped around the outer edge of a spool. You pull horizontally on the end of the ribbon so the ribbon starts to unwind from the spool as the spool rolls without slipping across a level surface.

(a) When you have moved the end of the ribbon through a horizontal distance L, how far has the spool moved?

(b) Does your answer change if the ribbon is instead wrapped around the spool's axle, which has a radius equal to half the radius of the spool? If so, how does the answer change?

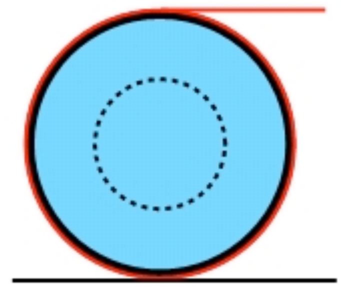

Figure 11.6: A spool is rolling without slipping to the right because you are pulling, to the right, on the red ribbon that is wrapped around the spool.

SOLUTION

(a) A diagram of the situation is shown in Figure 11.6. Once again, we can think of the spool's rolling motion as a combination of its translational motion and its rotational motion. We can thus say that the end of the ribbon moves because (a) the spool has a translational motion, and (b) the spool is rotating. The speed of the ribbon matches the speed of the top of the spool, because there is no slipping between the ribbon and the spool. Recalling the result from Exploration 11.3, the top of the spool has a velocity twice that of the center of the spool. Putting these facts together means that the center of the spool has a velocity half that of the end of the ribbon at any instant, and so the spool covers a distance of $L/2$, half the distance covered by the end of the ribbon.

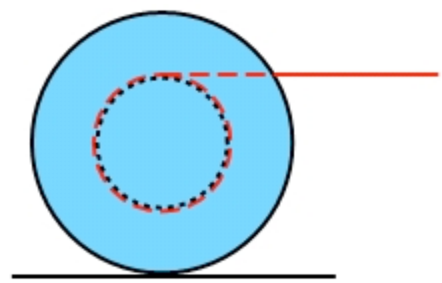

Figure 11.7: The ribbon is wrapped around the axle of the spool, which has a radius half that of the spool. The ribbon comes off the axle at the top.

(b) What if the ribbon is wrapped around the spool's axle and you move the end of the ribbon through a distance L? The answer changes because the rotational contribution to the net velocity changes. As shown in Figure 11.7, the ribbon now comes off the axle at the top of the axle, at a point halfway between the edge and the center of the spool. The net velocity at that point on the spool is 1.5 times the velocity of the center of the spool: the translational velocity is equal to the velocity of the center, while the rotational velocity is half that of the center, because at a radius of R/2 we have:

$$v_{rot} = \frac{R}{2}\omega = \frac{1}{2}R\omega = \frac{1}{2}v_{trans} .$$

Putting it another way, the velocity of the center of the spool is now two-thirds of the velocity of the end of the ribbon. If the end of the ribbon travels a distance L, the spool translates through a distance of $2L/3$.

Related End-of-Chapter Exercises: 18, 19.

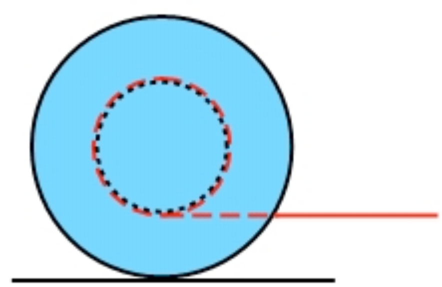

Figure 11.8: A ribbon is wrapped around the axle of the spool so the ribbon comes off the axle below the axle.

Essential Question 11.3: In a situation similar to that in Figure 11.7, you pull to the right on a ribbon wrapped around the axle of a spool. This time, however, the ribbon is wound so it comes away from the spool underneath the axle, as shown in Figure 11.8. When you pull to the right on the ribbon, the spool rolls without slipping. In which direction does it roll? Sketch a free-body diagram of the spool to help you think about this.