Answer to Essential Question 11.1: The effect of the change would be to decrease the rotational inertia of the pulley, because the rotational inertia of a solid sphere is $0.4MR^2$ compared with $0.5MR^2$ for the disk. The smaller the rotational inertia of the pulley, the less the pulley holds back the block, so the block's acceleration would increase. On the other hand, the force of tension would decrease. This is most easily seen by analyzing the block. If the block's acceleration increases, the net force on the block must increase. The force of gravity acting on the block remains constant, so the only way to increase the net force acting down on the block is to decrease the upward force of tension.

11-2 A General Method, and Rolling without Slipping

Let's begin by summarizing a general method for analyzing situations involving Newton's second law for rotation, such as the situation in Exploration 1.1. We will then explore rolling. We will tie together the two themes of this section in sections 11-3 and 11-4.

A General Method for Solving a Newton's Second Law for Rotation Problem

These problems generally involve both forces and torques.

1. Draw a diagram of the situation.
2. Draw a free-body diagram showing all the forces acting on the object.
3. Choose a rotational coordinate system. Pick an appropriate axis to take torques about, and then apply Newton's second law for rotation ($\sum \vec{\tau} = I\vec{\alpha}$) to obtain a torque equation.
4. Choose an appropriate x-y coordinate system for forces. Apply Newton's second law ($\sum \vec{F} = m\vec{a}$) to obtain one or more force equations. The positive directions for the rotational and x-y coordinate systems should be consistent with one another.
5. Combine the resulting equations to solve the problem.

Rolling without Slipping

Let's now examine a rolling wheel, which could be a bicycle wheel or a wheel on a car, truck, or bus. We will focus on a special kind of rolling, called **rolling without slipping**, in which the object rolls across a surface without slipping on that surface. This is actually what most rolling situations are, although our analysis would not apply to situations such as you spinning your car wheels on an icy road. Let's consider various aspects of rolling without slipping.

When we dealt with projectile motion in Chapter 4, we generally split the motion into two components, which were usually horizontal and vertical. To help understand rolling, we will follow a similar process. Rolling can be viewed as a combination, or superposition, of purely translational motion (moving a wheel from one place to another with no rotation) and purely rotational motion (only rotation with no movement of the center of the wheel). In the special case of rolling without slipping, there is a special connection between the translational component of the motion and the rotational component. Let's explore that connection.

EXPLORATION 11.2 – Rolling, rolling, rolling

We have a wheel of radius R that we will roll across a horizontal floor so that the wheel makes exactly one revolution. The wheel rolls without slipping on the floor.

Step 1 – ***Consider the rotational part of the motion only (focus on the fact that the wheel spins around exactly once). What distance does a point on the outer edge of the wheel travel because of this spinning motion?***

Because we're ignoring the rotational motion, the distance traveled by a point on the outer edge of the wheel because of the spin is equal to the circumference of the wheel itself. This is a distance of $2\pi R$. See the top diagram in Figure 11.4.

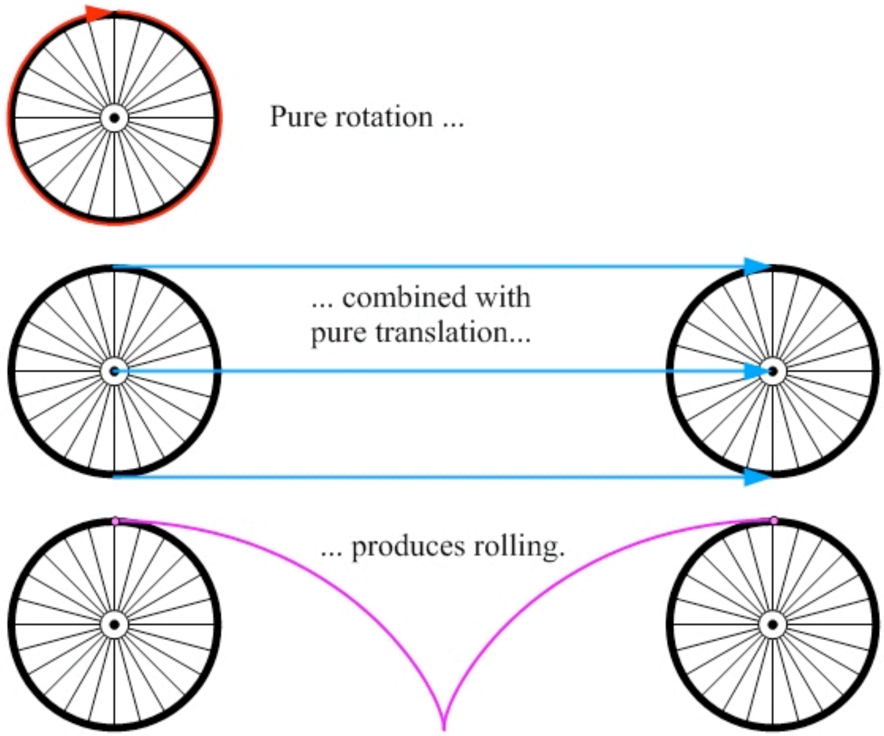

Figure 11.4: A pictorial representation of how the rotational component and the translational component of the motion combine to produce the interesting shape of the path traced out by a point on the outer edge of the wheel that is rolling without slipping. This shape is known as a cycloid.

Step 2 – ***Now consider the translational part of the motion only (i.e., ignore the fact that the wheel is spinning, and imagine that we simply drag the wheel a particular distance without allowing the wheel to rotate). What is the distance that any point on the wheel moves if we drag the wheel a distance equal to that it would move if we rolled it so it rolled through exactly one revolution?*** To determine what this distance is, imagine that we placed some double-sided tape around the wheel before we rolled it, and that the tape sticks to the floor. This is shown in the middle diagram in Figure 11.4. Rolling the wheel through one revolution lays down all the tape on the floor, covering a distance that is again equal to the circumference of the wheel. Thus, focusing on the translational distance only, the translational distance moved by every point on the wheel as the wheel rolls through one revolution is $2\pi R$.

Step 3 – ***Assuming the rolling is done at constant speed, compare the speed of a point on the outer rim, associated only with the wheel's rotation, to the translational speed of the wheel's center of mass.*** We can find these speeds by dividing the appropriate distances by the time during which the motion takes place. Because the distances associated with the two components of the motion are equal, and the time of the motion is the same for the two components, these two speeds are equal.

Key ideas for rolling: Rolling can be considered to be a superposition of a pure translational motion and a pure rotational motion. In the special case of rolling without slipping, the distance moved by a point on the outer edge of a wheel associated with the rotational component is equal to the translational distance of the wheel. The speed of a point on the outer edge because of the rotational component is also equal to the translational speed of the wheel.
Related End-of-Chapter Exercises: 4, 17.

Essential Question 11.2: Different points on a wheel that is rolling without slipping have different speeds. Considering one particular instant, which point on the wheel is moving slowest? Which point is moving the fastest?