## End-of-Chapter Exercises

Exercises 1 – 12 are conceptual questions that are designed to see if you have understood the main concepts of the chapter.

1. Figure 11.20 shows four different cases involving a uniform rod of length L and mass M is subjected to two forces of equal magnitude. The rod is free to rotate about an axis that either passes through one end of the rod, as in (a) and (b), or passes through the middle of the rod, as in (c) and (d). The axis is marked by the red and black circle, and is perpendicular to the page in each case. This is an overhead view, and we can neglect any effect of the force of gravity acting on the rod. Rank these four situations based on the magnitude of their angular acceleration, from largest to smallest.
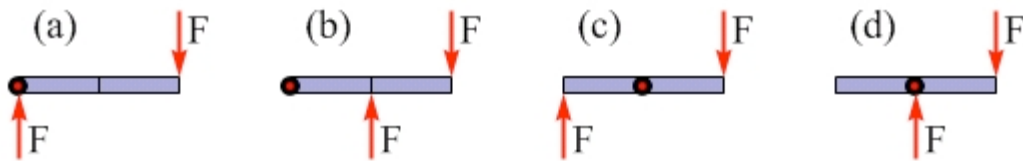

Figure 11.20: Four situations involving a uniform rod that can rotate about an axis being subjected to two forces of equal magnitude. For Exercise 1.

2. A pulley has a mass M, a radius R, and is in the form of a uniform solid disk. The pulley can rotate without friction about a horizontal axis through its center. As shown in Figure 11.21, the string wrapped around the outside edge of the pulley is subjected to an 8.0 N force in case 1, while, in case 2, a block with a weight of 8.0 N hangs down from the string. In which case is the angular acceleration of the pulley larger? Briefly justify your answer.
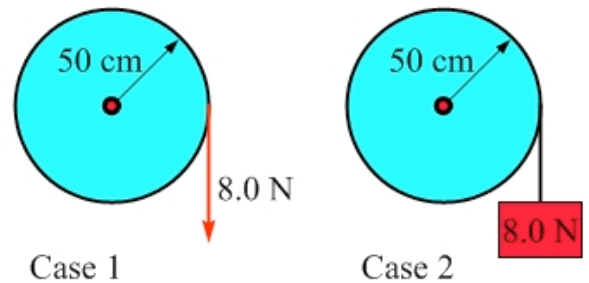
Figure 11.21: A frictionless pulley with a string wrapped around its outer edge. The string is subjected to an 8.0 N force in case 1, while in case 2 a block with a weight of 8.0 N hangs from the end of the string. For Exercise 2.

3. Consider again the spool shown in Figure 11.22, which we examined in Essential Question 11.2. In Essential Question 11.2, the spool rolled without slipping when a force to the right was exerted on the end of the string, but in this case let's say there is no friction between the spool and the horizontal surface. (a) Does the spool move? If so, which way does it move? (b) Does the spool rotate? If so, which way does it rotate?
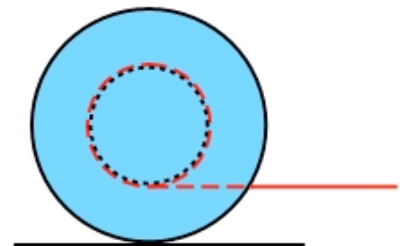
Figure 11.22: A spool with a ribbon wrapped around its axle. A force directed to the right is applied to the end of the ribbon. For Exercise 3.

4. In Exploration 11.2, we looked at how rolling motion can be viewed as a superposition of two simpler motions, pure translation and pure rotation. Have we done this before, broken down a more complicated motion into two simpler motions? If so, in what sort of situation? Comment on the similarities and differences between what we did previously and what we're doing in this chapter, for rolling.

5. A uniform solid cylinder rolls without slipping at constant velocity across a horizontal surface. In Exploration 11.3, we looked at how the net velocity of any point on such a rolling object can be determined. Is there any point on the cylinder with a net velocity directed in exactly the opposite direction as the cylinder's translational velocity? Briefly justify your answer.

6. You take a photograph of a bicycle race. Later, when you get home and look at the photo, you notice that some parts of each bicycle wheel in your photo are blurred, while others are not, or not as badly blurred. Compare the sharpness of the center of a wheel, the top of a wheel, and the bottom of a wheel. (a) Which point do you expect to be the most blurred? Why? (b) Which point do you expect to be in the sharpest focus? Why?

7. You have a race between two objects that have the same mass and radius by rolling them, without slipping, up a ramp. One object is a uniform solid sphere while the other is a ring. (a) Sketch a free-body diagram for one of the objects as it rolls without slipping up the ramp. (b) If the two objects have the same velocity at the bottom of the ramp, which object rolls farther up the ramp before turning around? Briefly justify your answer.

8. Repeat part (b) of the previous exercise, but this time the objects have the same total kinetic energy at the bottom of the ramp.

9. A figure skater is whirling around with her arms held out from her body. (a) What happens to her angular speed when she pulls her arms in close to her body? Why? (b) What happens to the skater's kinetic energy in this process? Explain your answer.

10. A solid cylinder is released from rest at the top of the ramp, and the cylinder rolls without slipping down the ramp. Defining the zero for gravitational potential energy to be at the level of the bottom of the ramp, the cylinder has a gravitational potential energy of 24 J when it is released. Draw a set of energy bar graphs to represent the cylinder's gravitational potential energy, translational kinetic energy, rotational kinetic energy, and total mechanical energy when the cylinder is (a) at the top of the ramp; (b) halfway down the ramp; and (c) at the bottom of the ramp.

11. Repeat Exercise 10, replacing the solid cylinder by a ring.

12. You have a race between a uniform solid sphere and a basketball, by releasing both objects from rest at the top of an incline. Which object reaches the bottom of the ramp first, assuming they both roll without slipping? Justify your answer.

Exercises 13 – 16 are designed to give you practice solving typical Newton's second law for rotation problems. For each exercise start with the following steps: (a) draw a diagram; (b) draw one or more free-body diagrams, as appropriate; (c) choose an appropriate rotational coordinate system and apply Newton's second law for rotation; (d) apply Newton's second law.

13. A block with a mass of 500 g is at rest on a frictionless table. A horizontal string tied to the block passes over a pulley mounted on the edge of the table, and the end of the string hangs down vertically below the pulley. The pulley is a uniform solid disk with a mass of 2.0 kg and a radius of 20 cm that rotates with no friction about an axis through its center. You then exert a constant force of 4.0 N down on the end of the string. The goal is to determine the acceleration of the block. Use $g = 10$ m/s². Parts (a) – (d) as described above. (e) Find the block's acceleration.

14. Repeat the previous exercise, but now there is some friction between the block and the table. The coefficient of static friction is $\mu_S = 0.40$, while the coefficient of kinetic friction is $\mu_K = 0.30$.

15. A 2.0-m long board is placed with one end on the floor and the other resting on a box that has a height of 30 cm. A uniform solid sphere is released from rest from the higher end of this board and rolls without slipping to the lower end. The goal is to determine how long the sphere takes to move from one end of the board to the other. Parts (a) – (d) as described above. (e) Combine your equations to determine the sphere's acceleration. (f) How long does it take the sphere to reach the lower end of the board?

16. A uniform solid cylinder with a mass of 2.0 kg rests on its side on a horizontal surface. A ribbon is wrapped around the outside of the cylinder with the end of the ribbon coming away from the cylinder horizontally from its highest point. When you exert a constant force of 4.0 N on the cylinder, the cylinder rolls without slipping in the direction of the force. The goal of this exercise is to determine the cylinder's acceleration. Parts (a) – (d) as described above. (e) Find the magnitude and direction of the force of friction exerted on the cylinder. (f) Find the cylinder's acceleration.

Exercises 17 – 21 deal with rolling situations.

17. A particular wheel has a radius of 50 cm. It rolls without slipping exactly half a rotation. (a) What is the translational distance moved by the wheel? (b) Considering motion due to the wheel's rotation only, what is the distance traveled by a point on the outer edge of the wheel? (c) Consider now the magnitude of the total displacement experienced by the point on the outer edge of the wheel that started at the top of the wheel. Is this equal to the sum of your two answers from (a) and (b)? Explain why or why not. (d) Work out the magnitude of the displacement of the point referred to in (c).

18. One end of a 2.0-m long board rests on a cylinder that has been placed on its side on a horizontal surface. You hold the other end of the board so the board is horizontal. When you walk forward, the cylinder rolls so the board does not slip on the cylinder, and the cylinder also rolls without slipping across the floor. When you move forward 1.0 m, how far does the cylinder move?

19. A particular point on a wheel is halfway between the center and the outer edge. When the point is at the same distance from the ground as the center of the wheel, the point's speed is 25.2 m/s. If the wheel has a radius of 35.0 cm and is rolling without slipping, find the translational speed of the wheel.

20. A solid sphere is released from rest and rolls without slipping down a ramp inclined at 12˚ to the horizontal. What is the sphere's speed when it is 1.0 m (measuring vertically) below the level it started?

21. Return to the situation described in Exercise 20, but this time the incline is changed to 6˚. The sphere again rolls without slipping starting from rest. Comparing the sphere's speed in both cases when it is 1.0 m (measuring vertically) below its starting point, in which case is the sphere moving faster? Rather then doing another calculation, see if you can come up with a conceptual argument to justify your answer.

Exercises 22 – 31 are modeled after similar exercises in previous chapters. Note the similarities between how we analyze rotational situations and how we analyze straight-line motion situations.

22. Two uniform solid disks, *A* and *B*, are initially at rest. The mass of disk *B* is two times larger than that of disk *A*. Identical net torques are then applied to the two disks, giving them each an angular acceleration as they rotate about their centers. Each net torque is removed once the object it is applied to has rotated through two revolutions. After both net torques are removed, how do: (a) the kinetic energies compare? (b) the angular speeds compare? (c) the angular momenta compare? (Compare this to Exercise 9 in Chapter 6.)

23. Repeat Exercise 22, assuming that both net torques are removed after the same amount of time instead. (Compare this to Exercise 10 in Chapter 6.)

24. Two identical grinding wheels of mass *m* and radius *r* are spinning about their centers. Wheel A has an initial angular speed of ω, while wheel B has an initial angular speed of 2ω. Both wheels are being used to sharpen tools. As shown in Figure 11.23, in both cases the tool is being pressed against the wheel with a force *F* directed toward the center of the wheel, and the coefficient of kinetic friction between the wheel and the tool is μ_K.

The tool does not move from the position shown in the diagram. (a) If it takes wheel A a time *T* to come to a stop, how long does it take for wheel B to come to a stop? (b) Find an expression for *T* in terms of the variables specified in the exercise. (c) If wheel A rotates through an angle θ before coming to rest, through what angle does wheel B rotate before coming to rest? (d) Find an expression for θ in terms of the variables specified in the exercise. (Compare this to Exercise 52 in Chapter 6.)
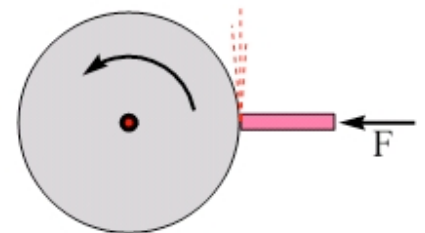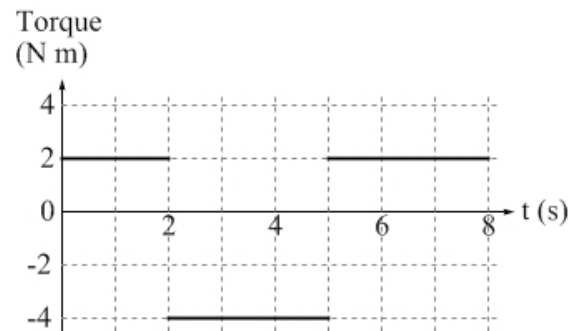
Figure 11.23: Sharpening a tool by holding it against a grinding wheel, for Exercises 24 – 26.

25. Return to the situation described in Exercise 24. How would *T*, the stopping time for wheel A, change if (a) *m* was doubled? (b) ω was doubled? (c) μ_K was doubled? (Compare this to Exercise 53 in Chapter 6.)

26. Return to the situation described in Exercise 24. How would θ, the angle wheel A rotates through before stopping, change if (a) *m* was doubled? (b) ω was doubled? (c) μ_K was doubled? (Compare this to Exercise 54 in Chapter 6.)

27. You pick up a bicycle wheel, with a mass of 800 grams and a radius of 40 cm, and spin it so the wheel rotates about its center. Assume that the mass of the wheel is concentrated in the rim. The initial angular speed is 5.0 rad/s, but after 10 s the angular speed is 3.0 rad/s. The goal here is to determine the magnitude of the frictional torque acting to slow the wheel, assuming it to be constant. (a) Sketch a diagram of the situation. (b) Choose a positive direction, and show this on the diagram. (c) Draw a free-body diagram of the wheel, focusing on the torque(s) acting on the wheel. (d) Write an expression for the net torque acting on the wheel. (e) Write an expression representing the wheel's change in angular momentum over the 10-second period. (f) Use the equation $\vec{\tau} \, \Delta t = \Delta \vec{L}$ to relate the expressions you wrote down in parts (d) and (e). (g) Solve for the frictional torque acting on the wheel. (Compare this to Exercise 23 in Chapter 6.)

28. At a time $t = 0$, a bicycle wheel with a mass of 4.00 kg has an angular velocity of 5.00 rad/s directed clockwise. For the next 8.00 seconds it then experiences a net torque, as shown in the graph in Figure 11.24 (taking clockwise to be positive). The wheel rotates about its center, and we can treat the wheel as a ring with a radius of $\dfrac{1}{\sqrt{2}}$ m. (a) Sketch a graph of the wheel's angular momentum as a function of time. (b) What is the cart's maximum angular speed during the 8.00-second interval the varying torque is being applied? At what time does the cart reach this maximum speed? (c) What is the cart's minimum angular speed during the 8.00-second interval the varying torque is being applied? At what time does the cart reach this minimum speed? (Compare this to Exercise 24 in Chapter 6.)

Figure 11.24: A plot of the torque applied to a bicycle wheel as a function of time, for Exercise 28.

29. Two blocks are connected by a string that passes over a frictionless pulley, as shown in Figure 11.25. Block A, with a mass $m_A = 2.0$ kg, rests on a ramp measuring 3.0 m vertically and 4.0 m horizontally. Block B hangs vertically below the pulley. The pulley has a mass of 1.0 kg, and can be treated as a uniform solid disk that rotates about its center. Note that you can solve this exercise entirely using forces and the constant-acceleration equations, but see if you can apply energy ideas instead. Use $g = 10$ m/s². When the system is released from rest, block A accelerates up the slope and block B accelerates straight down. When block B has fallen through a height $h = 2.0$ m, its speed is $v = 6.0$ m/s. (a) At any instant in time, how does the speed of block A compare to that of block B? (b) Assuming there is no friction acting on block A, what is the mass of block B? (Compare this to Exercise 44 in Chapter 7.)
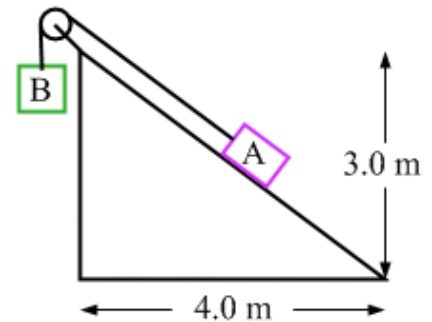
Figure 11.25: Two blocks connected by a string passing over a pulley, for Exercises 29 and 30.

30. Repeat Exercise 29, this time accounting for friction. If the coefficient of kinetic friction for the block A – ramp interaction is 0.625, what is the mass of block B? (Compare this to Exercise 45 in Chapter 7.)

31. A uniform solid sphere of mass m is released from rest at a height h above the base of a loop-the-loop track, as shown in Figure 11.26. The loop has a radius R. What is the minimum value of h necessary for the sphere to make it all the way around the loop without losing contact with the track? Express your answer in terms of R, and assume that the sphere's radius is much smaller than the loop's. (Compare this to Exercise 49 in Chapter 7.)
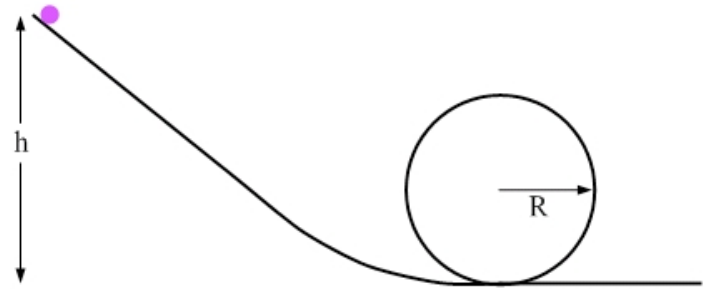
Figure 11.26: A solid sphere released from rest from a height h above the bottom of a loop-the-loop track, for Exercise 31.

Exercises 32 – 34 deal with angular momentum conservation.

32. A beetle with a mass of 20 g is initially at rest on the outer edge of a horizontal turntable that is also initially at rest. The turntable, which is free to rotate with no friction about an axis through its center, has a mass of 80 g and can be treated as a uniform disk. The beetle then starts to walk around the edge of the turntable, traveling at an angular velocity of 0.060 rad/s clockwise **with respect to the turntable**. (a) Qualitatively, what does the turntable do while the beetle is walking? Why? (b) With respect to you, motionless as you watch the beetle and turntable, what is the angular velocity of the beetle? What is the angular velocity of the turntable? (c) If a mark was placed on the turntable at the beetle's starting point, how long does it take the beetle to reach the mark? (d) Upon reaching the mark, the beetle stops. What does the turntable do? Why?

33. A bullet with a mass of 12 g is fired at a wooden rod that hangs vertically down from a pivot point that passes through the upper end of the rod. The bullet embeds itself in the lower end of the rod and the rod/bullet system swings up, reaching a maximum angular displacement of 60° from the vertical. The rod has a mass of 300 g, a length of 1.2 m, and we can assume the rod rotates without friction about the pivot point. What is the bullet's speed when it hits the rod? Assume the bullet is traveling horizontally when it hits the rod, and use $g = 10$ m/s².

34. A particular horizontal turntable can be modeled as a uniform disk with a mass of 200 g and a radius of 20 cm that rotates without friction about a vertical axis passing through its center. The angular speed of the turntable is 2.0 rad/s. A ball of clay, with a mass of 40 g, is dropped from a height of 35 cm above the turntable. It hits the turntable at a distance of 15 cm from the middle, and sticks where it hits. Assuming the turntable is firmly supported by its axle so it remains horizontal at all times, find the final angular speed of the turntable-clay system.

Use conservation of energy to solve Exercises 35 – 38. For each exercise begin by (a) writing down the energy conservation equation and choosing a zero level for gravitational potential energy; (b) identifying the terms that are zero and eliminating them; (c) writing out expressions for the remaining terms, remembering to account for both translational kinetic energy and rotational kinetic energy.

35. A uniform solid disk is released from rest at the top of a ramp, and rolls without slipping down the ramp. The goal of the exercise is to determine the disk's speed when it reaches a level 50 cm below (measured vertically) its starting point. Parts (a) – (c) as described above. (d) What is that speed?

36. The pulley shown in Figure 11.27 has a mass $M = 2.0$ kg and radius $R = 50$ cm, and can be treated as a uniform solid disk that can rotate about its center. The block (which has a mass of 800 g) hanging from the string wrapped around the pulley is then released from rest. The goal of the exercise is to determine the speed of the block when it has dropped 1.0 m. Parts (a) – (c) as described above. (d) What is the block's speed after dropping through 1.0 m?
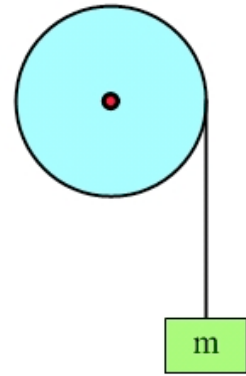
Figure 11.27: A block and a pulley, for Exercise 36.

37. A uniform solid sphere with a mass $M = 1.0$ kg and radius $R = 40$ cm is mounted on a frictionless vertical axle that passes through the center of the sphere. The sphere is initially at rest. You then pull on a string wrapped around the sphere's equator, exerting a constant force of 5.0 N. The string unwraps from the sphere when you have moved the end of the string through a distance of 2.0 m. The goal of the exercise is to determine the resulting angular speed of the sphere. Parts (a) – (c) as described above. (d) What is the resulting angular speed?

38. A uniform solid sphere with a mass of $M = 1.6$ kg and radius $R = 20$ cm is rolling without slipping on a horizontal surface at a constant speed of 2.1 m/s. It then encounters a ramp inclined at an angle of 10° with the horizontal, and proceeds to roll without slipping up the ramp. The goal of this exercise is to determine the distance the sphere rolls up the ramp (measured along the ramp) before it turns around. Parts (a) – (c) as described above. (d) How far does the sphere roll up the ramp? (e) Which of the values given in this exercise did you not need to find the solution?

General Problems and Conceptual Questions.

39. Figure 11.28 shows four different cases involving a uniform rod of length L and mass M is subjected to two forces of equal magnitude. The rod is free to rotate about an axis that either passes through one end of the rod, as in (a) and (b), or passes through the middle of the rod, as in (c) and (d). The axis is marked by the red and black circle, and is perpendicular to the page in each case. This is an overhead view, and we can neglect any effect of the force of gravity acting on the rod. If the rod has a length of 1.0 m, a mass of 3.0 kg, and each force has a magnitude of 5.0 N, determine the magnitude and direction of the angular acceleration of the rod in (a) Case (a); (b) Case (b); (c) Case (c); (d) Case (d).
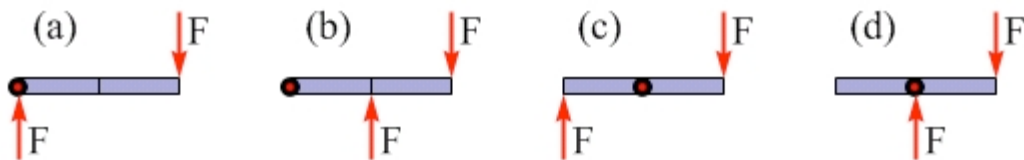
Figure 11.28: Four situations involving a uniform rod that can rotate about an axis being subjected to two forces of equal magnitude. For Exercise 39.

40. A pulley has a mass M, a radius R, and is in the form of a uniform solid disk. The pulley can rotate without friction about a horizontal axis through its center. As shown in Figure 11.29, the string wrapped around the outside edge of the pulley is subjected to an 8.0 N force in case 1, while in case 2 a block with a weight of 8.0 N hangs down from the string. If $M = 2.0$ kg and $R = 50$ cm, calculate the angular acceleration of the pulley in (a) case 1; (b) case 2.

50 cm

8.0 N

Case 1

50 cm

8.0 N
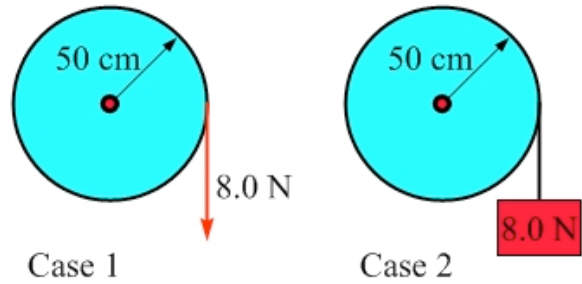
Case 2

Figure 11.29: A frictionless pulley with a string wrapped around its outer edge. The string is subjected to an 8.0 N force in case 1, while in case 2 a block with a weight of 8.0 N hangs from the end of the string. For Exercise 40.

41. Atwood's machine is a system consisting of two blocks that have different masses connected by a string that passes over a frictionless pulley, as shown in Figure 11.30. The pulley has a mass m_p. Compare the tension in the part of the string just above the block with the larger mass, M, to that in the part of the string just above the block with the smaller mass, m, in the following cases: (a) You hold on to the smaller-mass block to keep the system at rest; (b) The system is released from rest; (c) You hold on to the smaller-mass block and pull down so the blocks move with constant velocity. Justify your answers in each case.


m_p

M

m
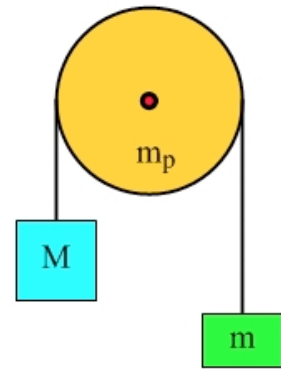
Figure 11.30: Atwood's machine – a device consisting of two objects connected by a string that passes over a pulley. For Exercises 41 – 43.

42. Atwood's machine is a system consisting of two objects connected by a string that passes over a frictionless pulley, as shown in Figure 11.30. In Chapter 5, we neglected the effect of the pulley, but now we know how to account for the pulley's impact on the system. (a) If the two objects have masses of M and m, with $M > m$, and the pulley is in the shape of a uniform solid disk and has a mass m_p, derive an expression for the acceleration of either block, in terms of the given masses and g. (b) What does the expression reduce to in the limit where the mass of the pulley approaches zero? (c) How does accounting for the fact that the pulley has a non-zero mass affect the magnitude of the acceleration of a block?

43. Consider again the Atwood's machine described in Exercise 42 and pictured in Figure 11.30. (a) If $M = 500$ g, $m = 300$ g, and the pulley mass is $m_p = 400$ g, what is the magnitude of the acceleration of one of the blocks? (b) If the system is released from rest, what is the angular velocity of the pulley 2.0 seconds after the motion begins? The pulley has a radius of 10 cm.

44. A particular double-pulley consists of a small pulley of radius 20 cm mounted on a large pulley of radius 50 cm, as shown in Figure 11.31. A block of mass 2.0 kg hangs from a string wrapped around the large pulley. To keep the system at rest, what mass block should be hung from the small pulley?
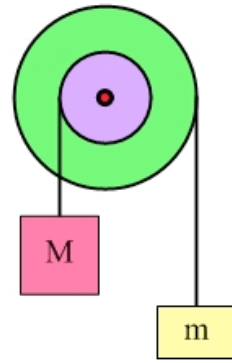
45. A particular double pulley consists of a small pulley of radius 20 cm mounted on a large pulley of radius 50 cm, as shown in Figure 11.31. The pulleys rotate together, rather than independently. A block of mass 2.0 kg hangs from a string wrapped around the large pulley, while a second block of mass 2.0 kg hangs from the small pulley. Each pulley has a mass of 1.0 kg and is in the form of a uniform solid disk. Use $g = 9.8$ m/s². (a) What is the acceleration of the block attached to the large pulley? (b) What is the acceleration of the block attached to the small pulley?

Figure 11.31: A double-pulley system, with a 2.0 kg block hung from the string wrapped around the large pulley and a second block hung from the string wrapped around the small pulley. For Exercises 44 – 46.

46. A pulley consists of a small uniform disk of radius 0.50 m mounted on a larger uniform disk of radius 1.0 m. Each disk has a mass of 1.0 kg. The pulley can rotate without friction about an axis through its center. As shown in Figure 11.31, a block with mass $m = 1.0$ kg hangs down from the larger disk while a block of mass M hangs down from the smaller disk. If the angular acceleration of the system has a magnitude of 1.0 rad/s² what is the value of M? Consider all possible answers, and use $g = 10$ m/s².

47. A yo-yo consists of two identical disks, each with a mass of 40 g and a radius of 4.0 cm, joined by a small cylindrical axle with negligible mass and a radius of 1.0 cm. When the yo-yo is released it essentially rolls without slipping down the string wrapped around the axle. If the end of the string (the one you would hold) remains fixed in place, determine the acceleration of the yo-yo. Use $g = 9.8$ m/s².
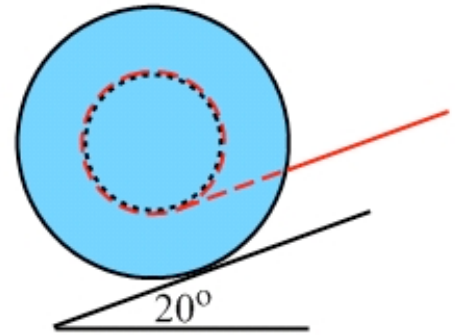
48. As shown in Figure 11.32, blocks A and B are connected by a massless string that passes over the outer edge of a pulley that is a uniform solid disk. The mass of block A is equal to that of block B; the mass of the pulley, coincidentally, is also the same as that of block A. When the system is released from rest it experiences a constant (and non-zero) acceleration. There is no friction between block A and the surface. Use $g = 10.0$ m/s². (a) What is the acceleration of the system? (b) The two parts of the string have different tensions. In which part of the string is the tension larger, between block A and the pulley or between the pulley and block B? Briefly justify your answer. (c) If the tension in one part of the string is 3.00 N larger than the tension in the other part, what are the values of the tensions in the two parts of the string?
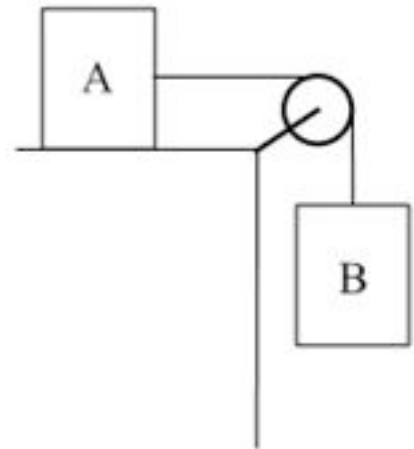
Figure 11.32: Two blocks connected by a string passing over a pulley, for Exercises 48 and 49.

49. Repeat Exercise 48, but this time there is some friction between block A and the surface, with a coefficient of kinetic friction of 0.500.

50. A spool has a string wrapped around its axle, with the string coming away from the underside of the spool. The spool is on a ramp inclined at 20° with the horizontal, as shown in Figure 11.33. There is no friction between the spool and the ramp. Assuming you can exert as much or as little force on the end of the string as you wish (always directed up the slope) which of the following situations are possible? If so, explain how the situation could be achieved; if not, explain why not. (a) The spool remains completely motionless. (b) The spool rotates about its center but does not move up or down the ramp. (c) The spool has no rotation but moves down the ramp.

51. Return to the situation described in Exercise 50 and shown in Figure 11.33, but now there is friction between the spool and the ramp. Is it possible for the spool to remain completely motionless now? If so, explain how. If not, explain why not.

Figure 11.33: A spool on a ramp inclined at 20° with the horizontal. The string wrapped around the spool's axle comes away from the spool on its underside. Any force exerted on the end of the string is exerted up the slope. For Exercises 50 and 51.

52. Consider the spool shown in Figure 11.34. The spool has a radius R, while the spool's axle has a radius of $R/2$. There is some friction between the spool and the horizontal surface it is on, so that when a modest tension is exerted on the string the spool may roll one way or the other without slipping. It turns out that when the angle θ between the string and the vertical is larger than some critical value θ_C, the spool rolls without slipping one way; when $\theta < \theta_C$, the spool rolls the other way, and when $\theta = \theta_C$, the spool remains at rest. (a) Find θ_C. (b) Which way does the spool roll when $\theta < \theta_C$?
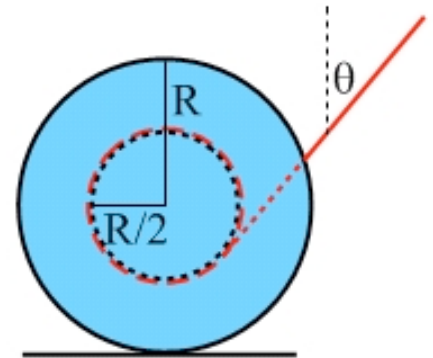
Figure 11.34: A spool on a horizontal surface, for Exercise 52.

53. A spool consists of two disks, each of radius R and mass M, connected by a cylindrical axle of radius $R/2$ and mass M. When an upward force of magnitude F is exerted on a string wrapped around the axle, the spool will roll without slipping as long as F is not too large. (a) What is the spool's rotational inertia, in terms of M and R, about an axis through its center? (b) If the coefficient of static friction between the spool and the horizontal surface it is on is 0.50, what is the maximum value F can be, in terms of M and g, for the spool to roll without slipping?

54. A solid sphere rolls without slipping when it is released from rest at the top of a ramp that is inclined at 30° with respect to the horizontal, but, if the angle exceeds 30°, the sphere slips as it rolls. Calculate the coefficient of static friction between the sphere and the incline.

55. While fixing your bicycle, you remove the front wheel from the frame. A bicycle wheel can be approximated as a ring, with all the mass of the wheel concentrated on the wheel's outer edge. The wheel has a mass M, a radius R, and it is initially spinning at a particular angular velocity. There is a constant frictional torque that is causing the wheel to slow down, however. You also have a uniform solid disk of the same mass and radius as the bicycle wheel. It also has the same initial angular velocity and the same frictional torque as the wheel. Which of these objects will spin for the longer time? Justify your answer.

56. As shown in Figure 11.35, a bowling ball of mass M and radius $R = 20.0$ cm is released with an initial translational velocity of $\bar{v}_0 = 14.0$ m/s to the right and an initial angular velocity of $\bar{\omega}_0 = 0$. The bowling ball can be treated as a uniform solid sphere. The coefficient of kinetic friction between the ball and the surface is $\mu_K = 0.200$. The force of kinetic friction causes a linear acceleration, as well as a torque that causes the ball to spin. The ball slides along the horizontal surface for some time, and then rolls without slipping at constant velocity after that. **Use $g = 10$ m/s²**. (a) Draw the free-body diagram of the ball showing all the forces acting on it while it is sliding. (b) What is the acceleration of the ball while it is sliding? (c) What is the angular acceleration of the ball while it is sliding? (d) How far does the ball travel while it is sliding? (e) What is the constant speed of the ball when it rolls without slipping?
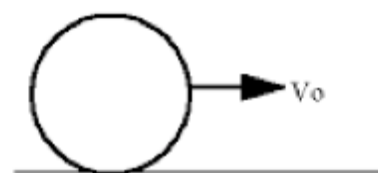
Figure 11.35: The initial state of a bowling ball, for Exercise 56.

57. Figure 11.36 shows the side view of a meter stick that can rotate without friction about an axis passing through one end. Pennies (of negligible mass in comparison to the mass of the meter stick) have been placed on the meter stick at regular intervals. When the meter stick is released from rest, it rotates about the axis. Some of the pennies remain in contact with the meter stick while some lose contact with it. (a) Which pennies do you expect to lose contact with the meter stick, the ones close to the axis or the ones farther from it? (b) Determine the initial acceleration of the right end of the meter stick, and of the center of the meter stick, to help justify your conclusion in (a).

Figure 11.36: A side view of a meter stick, with pennies resting on it at regular intervals, that is initially held horizontal. The meter stick can rotate without friction about an axis through the left end. For Exercises 57 and 58.

58. Return to the situation described in Exercise 57 and shown in Figure 11.36. Assuming the axis is at the 0-cm mark of the meter stick, determine the point on the meter stick beyond which the pennies will lose contact with the meter stick when the system is released from rest.

59. A particularly large playground merry-go-round is essentially a uniform solid disk of mass $4M$ and radius R that can rotate with no friction about a central axis. You, with a mass M, are a distance of $R/2$ from the center of the merry-go-round, rotating together with it at an angular velocity of 2.4 rad/s clockwise (when viewed from above). You then walk to the outside of the merry-go-round so you are a distance R from the center, still rotating with the merry-go-round. Consider you and the merry-go-round to be one system. (a) When you walk to the outside of the merry-go-round, does the angular momentum of the system increase, decrease, or stay the same? Why? (b) Does the kinetic energy of the system increase, decrease, or stay the same? Why? (c) If you started running around the outer edge of the merry-go-round, at what angular velocity would you have to run to make the merry-go-round alone come to a complete stop? Specify the magnitude and direction.

60. Consider the following situations. For each, state whether you would apply energy methods, torque/rotational kinematics methods, or either to solve the exercise. You don't need to solve the exercise. (a) Find the final speed of a uniform solid sphere that rolls without slipping down a ramp inclined at 8.0° with the horizontal, if the sphere is released from rest and the vertical component of its displacement is 1.0 m. (b) Find the time it takes the sphere in (a) to reach the bottom of the ramp. (c) Find the number of rotations the sphere in (a) makes as it rolls down the ramp, if the sphere's radius is 15 cm.

61. Return to Exercise 60, and this time solve each part.

62. The planet Earth orbits the Sun in an orbit that is roughly circular. Assuming the orbit is exactly circular, which of the following is conserved as the Earth travels along its orbit? (a) Its momentum? (b) Its angular momentum, relative to an axis passing through the center, and perpendicular to the plane, of the orbit? (c) Its translational kinetic energy? (d) Its gravitational potential energy? (e) Its total mechanical energy? For any that are not conserved, explain why.

63. A typical comet orbit ranges from relatively close to the Sun to many times farther than the Sun. Which of the following is conserved as the comet travels along its orbit? (a) Its angular momentum, relative to an axis passing through the Sun, and perpendicular to the plane, of the orbit? (b) Its translational kinetic energy? (c) Its gravitational potential energy? (d) Its total mechanical energy? For any that are not conserved, explain why.

64. Two of your classmates, Alex and Shaun, are carrying on a conversation about a physics problem. Comment on each of their statements.

Alex: In this situation, we have to draw the free-body diagram for a sphere that is rolling without slipping up an incline. OK, so, there's a force of gravity acting down, and a normal force perpendicular to the surface. Is there a friction force?

Shaun: Don't we need another force directed up the incline, in the direction of motion?

Alex: I don't think so. I think we just need to add a kinetic friction force down the incline, opposing the motion.

Shaun: Wait a second – isn't it static friction? Isn't it always static friction when something rolls without slipping?