

Answer to Essential Question 10.7: Yes, the rotational inertia of a system of objects can be found by adding up the rotational inertias of the various objects making up the system. This is precisely the process we followed in Example 10.7.

10-8 A Table of Rotational Inertias

Consider now what happens if we take an object that has its mass distributed over a length, area, or volume, rather than being concentrated in one place. Generally, the rotational inertia in such a case is calculated by breaking up an object into tiny pieces, finding the rotational inertia of each piece, and adding up the individual rotational inertias to determine the total rotational inertia.

We can get a feel for the process by considering how we would find the rotational inertia of a uniform rod of length L and mass M , rotating about an axis through the end of the rod that is perpendicular to the rod itself. If all the mass were concentrated at the far end of the rod, a distance L from the axis, then the rotational inertia would be ML^2 . Because most of the mass is closer than L to the axis of rotation, the rod's rotational inertia turns out to be less than ML^2 . If we broke up the rod into ten equal pieces, with centers at 5%, 15%, 25%, 35%, ..., 95% of the length of the rod (see Figure 10.22), we would calculate a rotational inertia of $0.3325 ML^2$. This is very close to the value we would get by doing the integration,

$$I_{\text{rod, end}} = ML^2 / 3. \text{ The rotational}$$

inertia's of various shapes, and for various axes of rotation, are shown in Figure 10.23.

Essential Question 10.8: In Figure 10.23, all the values for rotational inertia are of the form

$I = cMR^2$, or $I = cML^2$, where c is generally less than 1. The exception is the rotational inertia of a ring rotating about an axis through the center of the ring and perpendicular to the plane of the ring, where $c = 1$. Why do we expect to get $I = MR^2$ for the ring rotating about that central axis?

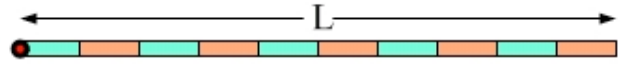
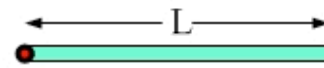
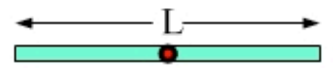


Figure 10.22: A uniform rod of length L and mass M , divided into 10 equal pieces. The axis of rotation passes through the left end of the rod and is perpendicular to the page.



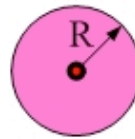
Rod rotating about an axis through one end, perpendicular to the rod.

$$I = \frac{1}{3}ML^2$$



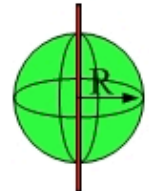
Rod rotating about an axis through the middle, perpendicular to the rod.

$$I = \frac{1}{12}ML^2$$



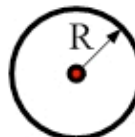
Solid disk or cylinder about an axis through the middle, perpendicular to the plane of the disk.

$$I = \frac{1}{2}MR^2$$



Solid sphere about an axis through the center.

$$I = \frac{2}{5}MR^2$$



Thin ring about an axis through the middle, perpendicular to the plane of the ring.

$$I = MR^2$$



Hollow sphere about an axis through the center.

$$I = \frac{2}{3}MR^2$$

Figure 10.23: Expressions for the rotational inertia of various objects about a particular axis. In each case, the object has a mass M .