

Answer to Essential Question 10.5: The torque, from the tension, is zero with respect to any axis that passes through the string, because the line of the force (the string, in this case) passes through an axis that lies on the string. It is important to remember that the torque (both its direction and magnitude) associated with a force depends on the particular axis of rotation the torque is being measured with respect to.

10-6 Rotational Inertia

In Chapter 3, we found that an object's acceleration is proportional to the net force acting on the object:

$$\vec{a} = \frac{\sum \vec{F}}{m}. \quad (\text{Equation 3.1: Connecting acceleration to net force})$$

A similar relationship connects the angular acceleration of an object to the net torque acting on it:

$$\vec{\alpha} = \frac{\sum \vec{\tau}}{I}. \quad (\text{Eq. 10.10: Connecting angular acceleration to net torque})$$

Thus, the angular acceleration of an object is proportional to the net torque acting on the object. The I in the denominator of Equation 10.10 is known as the rotational inertia, which is the rotational equivalent of mass.

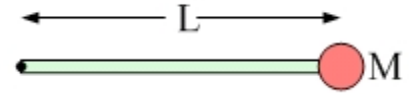
We have already looked at how the angular acceleration $\vec{\alpha}$ is the rotational equivalent of the acceleration \vec{a} , and how torque, $\vec{\tau}$, is the rotational equivalent of force, \vec{F} . The I in the denominator of Equation 10.10 must therefore be the rotational equivalent of the mass, m . I is known as the **rotational inertia**, or the **moment of inertia**. In the same way that mass is a measure of an object's tendency to maintain its state of straight-line motion, an object's rotational inertia is a measure of the object's tendency to maintain its rotational motion. Something with a large mass is hard to get moving, and it is also hard to stop if it is already moving. Similarly, if an object has a large rotational inertia it is difficult to start it rotating, and difficult to stop if it is already rotating.

One question to consider is, are rotational inertia and mass the same thing? In other words, does an object's mass, by itself, determine the rotational inertia? Let's check the units of rotational inertia. Re-arranging Equation 10.10, we find that rotational inertia has units of torque units (N m) divided by angular acceleration units (rad/s²). Remembering that the newton is equivalent to kg m/s², and that we can treat the radian as being dimensionless, we find that rotational inertia has units of kg m². Rotational inertia depends on more than just mass, it depends on both mass and, somehow, length squared. Let's investigate this further.

EXPLORATION 10.6 – Rotational inertia

Consider a ball of mass M mounted at the end of a stick that has a negligible mass, and a length L (which is large compared to the ball's radius). The other end of the stick is pinned so the stick can rotate freely about the pin.

Step 1 – If the ball and stick are held horizontal and then released from rest, what is the ball's initial acceleration? The ball's initial acceleration is \vec{g} , the acceleration due to gravity. The force of the stick acting on the ball only becomes non-zero after the ball starts moving. We should also draw a diagram to help analyze the situation. The diagram is shown in Figure 10.16.



Step 2 – What is the ball’s initial angular acceleration?

The angular acceleration can be found from the equation $a = r\alpha$. Here $r = L$, the length of the stick, so we have $\alpha = g/L$, directed clockwise.

Figure 10.16: The initial position of the ball and stick. The system can rotate about an axis passing through the left end of the stick.

Step 3 – What is the torque acting on the ball at the instant it is released?

Here we can draw a free-body diagram of the ball, shown in Figure 10.17. Initially the only force acting on the ball is the force of gravity, Mg directed down. Considering an axis perpendicular to the page and passing through the pin, the torque is $\tau = LMg$, directed clockwise.

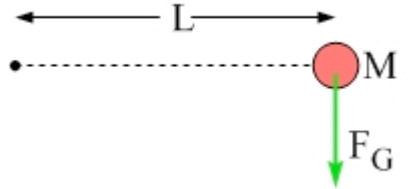


Figure 10.17: The free-body diagram of the ball immediately after the system is released from rest.

Step 4 – Using Equation 10.10, and the results from steps 2 and 3, determine the rotational inertia of the ball relative to the axis passing through the pin.

Re-arranging Equation 10.10 to solve for the rotational inertia gives:

$$I = \frac{\sum \tau}{\alpha}$$

The torque and the angular acceleration are both clockwise, allowing us to divide the magnitude of the torque by the magnitude of the angular acceleration to determine the ball’s rotational inertia about an axis through the pin.

$$I = \frac{LMg}{g/L} = ML^2$$

Thus the rotational inertia of an object of mass M in which all the mass is at a particular distance L from the axis of rotation is $I = ML^2$.

Key ideas for rotational inertia: An object’s rotational inertia is determined by three factors: the object’s mass; how the object’s mass is distributed; and the axis the object is rotating around.
Related End-of-Chapter Exercises: 10, 27.

Essential Question 10.6: Consider the three cases shown in Figure 10.18. In each case, a ball of a particular mass is placed on a light rod of a particular length. Each rod can rotate without friction about an axis through the left end. Rank the cases based on their rotational inertias, from largest to smallest.

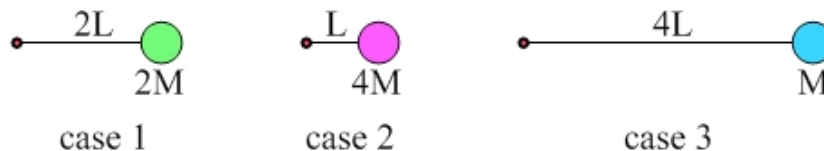


Figure 10.18: Three cases, each involving a ball on the end of a rod that can rotate about its left end.