Answer to Essential Question 10.3: Because the straight-line motion variables are related to the equivalent rotational variables by a factor of r (e.g., $v = r\omega$), changing the value of r requires changing the graphs for the straight-line motion by a factor equal to the numerical value of r. One way to do this is to draw the lines on each graph exactly as before, but re-scale each y-axis. In the case of r = 3.0 m, for instance, each number on each y-axis would be multiplied by a factor of 3.0. A second approach is to keep the scales on the axes the same as before but move the graphs. For instance, the graph of velocity vs. time, which is given by the equation $v = +(\pi / 3)\ \text{m/s}^2 \times t$ when r = 1.0 m, would be given by $v' = +(\pi\ \text{m/s}^2)\, t$ when r = 3.0 m.

10-4 Torque

If an object is at rest, how can we get it to rotate? If an object is already rotating, how can we change its rotational motion? We answered equivalent questions about straight-line motion by saying "Apply a net force!" Let's now consider the rotational equivalent of force.

EXPLORATION 10.4 – Turning a revolving door

From an overhead view, a revolving door looks like a + sign mounted on a vertical axle. The door can spin freely, clockwise or counterclockwise, about its center.

Step 1 – ***Consider the three cases illustrated in Figure 10.9, in which a force (the red arrow) is applied to a revolving door. In each case, determine the direction the door will start to rotate, assuming it starts from rest.***

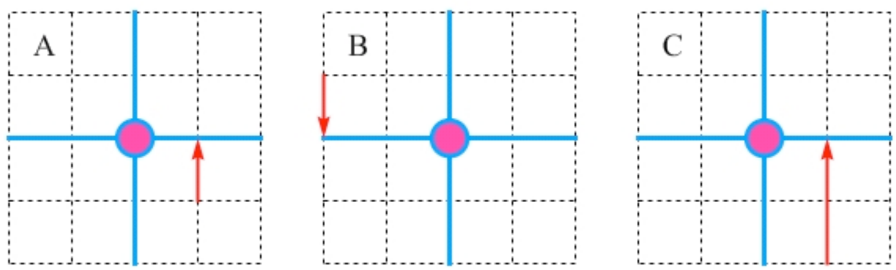

Figure 10.9: Three cases of forces applied to a revolving door, shown from an overhead perspective.

Although the direction of the force in case B is opposite to that in cases A and C, in each case the door will rotate counterclockwise. If you are ever confused about the direction an object will tend to rotate, place your pen or pencil on the diagram and hold it at the axis of the object, in this case at the center. Then push on the object in the direction, and at the location, of the applied force and see which way the object spins. Knowing the direction of a force applied to an object is not enough to determine the direction of rotation; we also need to know where the force is applied in relation to the axis of rotation.

Step 2 – ***Rank the three cases based on how quickly the revolving door spins, from largest to smallest, assuming the door is initially at rest.*** In case C, the door will rotate more quickly than in case A, because the applied force in C is twice as large as that in A while everything else (the point at which the force is applied, and the direction of the force) is equal. The door in case B also rotates faster than that in A because, even though the force has the same magnitude, in case B the force is applied further from the axis of rotation. Applying a force farther from the axis of rotation generally has a larger effect on the rotation of an object, which you have probably experienced. If you have ever come to a door where it was not obvious which side was connected to the hinges, and given the door a push on the edge where the hinges were, you most likely came close to running straight into the door as it opened very slowly in response to your push. Applying the same force at the edge of the door furthest from the hinges, however, is far more effective at opening the door.

The comparison that is hardest to rank is that between B and C. In case C the applied force is twice as large as that in B, but the force in B is applied twice as far from the axis of rotation as that in C. Which effect is more important? It turns out that these effects are equally important, so cases B and C are equivalent. The overall ranking is B=C>A.

The point of this discussion is that the angular acceleration of the door is proportional to both the applied force and the distance of the applied force from the axis of rotation. Let's now consider whether the direction at which the force is applied makes any difference.

Step 3 – *Consider the three cases shown in Figure 10.10. Rank these three cases based on the revolving door's angular acceleration, from largest to smallest.*

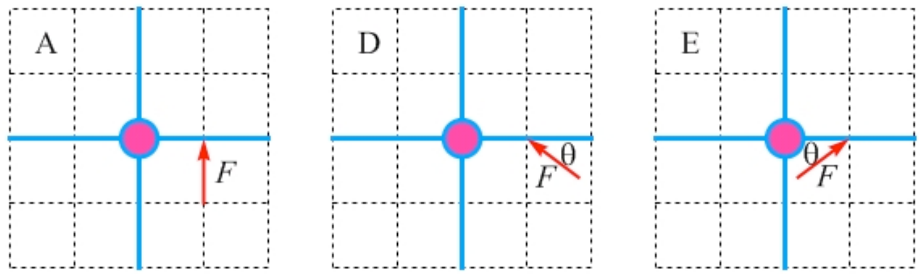

Figure 10.10: Three cases involving the same magnitude force applied at the same point on a revolving door, but applied in different directions.

Let's split the forces in cases D and E into components, as shown in Figure 10.11. How do the components of the force influence the door in each case? If you've ever tried to open a door by exerting a force parallel to the door itself, you'll know that this is completely ineffective. Similarly, the parallel components in cases D and E do absolutely nothing to affect the door's rotation. Only the perpendicular components, which have a magnitude of $F\sin\theta$, affect the rotation. Because these components are smaller than F, the magnitude of the perpendicular force in case A, ranking the three cases gives A>D=E.

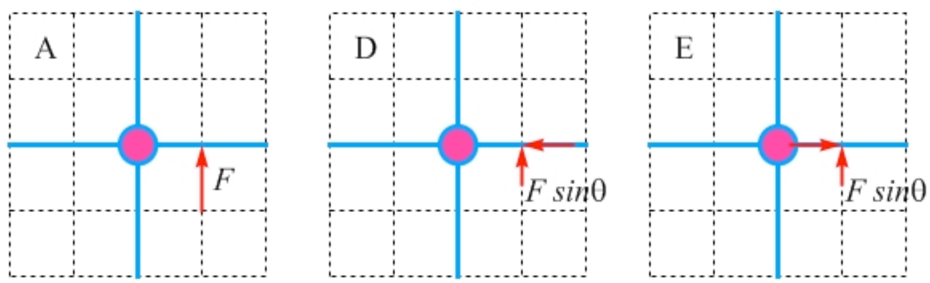

Figure 10.11: Splitting the force in case D, and case E, into components parallel to the door and perpendicular to the door.

Key ideas: The angular acceleration of a door depends on three factors: the magnitude of the applied force; the distance from the axis of rotation to where the force is applied; and the direction of the applied force. **Related End-of-Chapter Exercises: 48, 49.**

In Exploration 10.4, we learned about the rotational equivalent of force, which is torque.

The name for the rotational equivalent of force is **torque**, which we symbolize with the Greek letter tau (τ). Whereas a force is a push or a pull, a torque is a twist. A torque can result from applying a force. The torque resulting from applying a force F at a distance r from an axis of rotation is:

$$\tau = rF\sin\theta \; . \qquad \text{(Equation 10.9: \textbf{Magnitude of the torque})}$$

The angle θ represents the angle between the line of the force and the line the distance r is measured along.

Essential Question 10.4: Make a list of common household items or tools that exploit principles of torque.