Answer to Essential Question 10.2: Because Table 10.3 deals with angular variables, which are independent of the radius, nothing would change in the table.

10-3 Solving Rotational Kinematics Problems

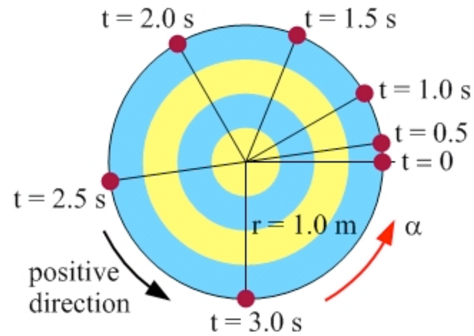

Figure 10.6: A motion diagram for an object moving with an accelerating turntable, showing the position at 0.5-second intervals.

EXPLORATION 10.3A – Unrolling the motion

Return to the situation from Example 10.2. Let's take the object's path, which is a circular arc (see Figure 10.5), and unroll it so it is a straight line. How would we analyze this straight-line motion? Unrolling the circular arc from Figure 10.6 gives the straight-line motion situation shown in Figure 10.7. Let's define the line the object moves along to be the *x*-axis. The origin is the object's initial position, and the positive direction is to the right. This situation should look familiar, because it is an excellent example of one-dimensional motion with constant acceleration, as we studied in Chapter 2.

For the rotational situation, the distance traveled is the length of the circular arc the object moves along. After unrolling the arc to get a straight line, we can use Equation 10.1, $s = r\theta$, to find the arc length corresponding to the distance traveled from the origin. When using this equation, use angles in radians. Table 10.4 builds on Table 10.3, bringing in a row for the arc length *s*, which is the same as the position, *x*, for the equivalent one-dimensional motion. Because we have the special case *r* = 1.0 m, *s* and *θ* are numerically equal, and differ only in their units.

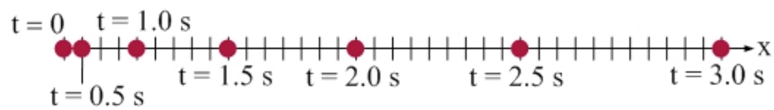

Figure 10.7: The straight-line motion resulting from straightening the circular arc traveled by the object in Figure 10.6.

Time (s)	0	0.50	1.00	1.50	2.00	2.50	3.00
Angular position, *θ* (rad)	0	$+\pi/24$	$+\pi/6$	$+3\pi/8$	$+2\pi/3$	$+25\pi/24$	$+3\pi/2$
***s* or *x* (m)**	0	$+\pi/24$	$+\pi/6$	$+3\pi/8$	$+2\pi/3$	$+25\pi/24$	$+3\pi/2$

Table 10.4: Determining the arc length, and the displacement in the corresponding 1-dimensional motion situation, for the object on the turntable.

Key idea: Rotational motion with constant angular acceleration is analogous to one-dimensional motion with constant acceleration. **Related End-of-Chapter Exercises: 2, 39.**

Based on Example 10.2 and Exploration 10.3A, let's write down a general method for solving rotational kinematics problems. The method parallels the method used in Chapter 2 for solving one-dimensional kinematics problems.

A General Method for Solving a Rotational Kinematics Problem

1. Draw a diagram of the situation.
2. Choose an origin to measure positions from, and mark it on the diagram.
3. Choose a positive direction, and mark this on the diagram with an arrow.
4. Organize what you know, and what you're looking for. Making a date table is a useful way to organize the information.
5. Think about which of the constant-acceleration equations to apply, and then set up and solve the problem. The three main equations are:

$$\omega = \omega_i + \alpha t .$$ (Equation 10.6)

$$\theta = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 .$$ (Equation 10.7)

$$\omega^2 = \omega_i^2 + 2\alpha \Delta\theta .$$ (Equation 10.8)

EXPLORATION 10.3B – Graphs for rotational motion

Plot a set of graphs showing, as a function of time, the angular acceleration, the angular velocity, and the angular position of the object on the turntable we considered in Exploration 10.3A. How does this set of graphs compare to graphs showing, as a function of time, the acceleration, velocity, and position of the equivalent straight-line motion situation that we considered in Exploration 10.3A?

The angular acceleration is constant, with a value of $+(\pi / 3) \text{ rad/s}^2$. The graph of the angular acceleration is the horizontal line shown at the top of Figure 10.8.

To graph angular velocity as a function of time, we can use Equation 10.6, $\omega = \omega_i + \alpha t$. Substituting values for the initial angular velocity and the angular acceleration gives: $\omega = 0 + (\pi / 3) \text{ rad/s}^2 \times t = (\pi / 3) \text{ rad/s}^2 \times t$.

This function is a straight line, starting from the origin, with a constant slope, as shown in the middle graph of Figure 10.8.

To graph the angular position as a function of time we can use Equation 10.7, as in Exploration 10.2, to get

$$\theta = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 = 0 + 0 + \left(\frac{\pi}{6} \text{rad/s}^2\right) t^2 = +\left(\frac{\pi}{6} \text{rad/s}^2\right) t^2 .$$

Recall that values of the angular position as a function of time are given in Table 10.3, and repeated in 10.4, so those points can be plotted on a graph and a smooth curve drawn through them. The result is the quadratic graph shown at the bottom of Figure 10.8.

Note that, because $r = 1.0$ m, we can actually use these same graphs to represent the acceleration, velocity, and position of the equivalent straight-line motion that we considered in the previous Exploration. We would need to change the units and the labels on the three y-axes, but the graphs would otherwise look identical.

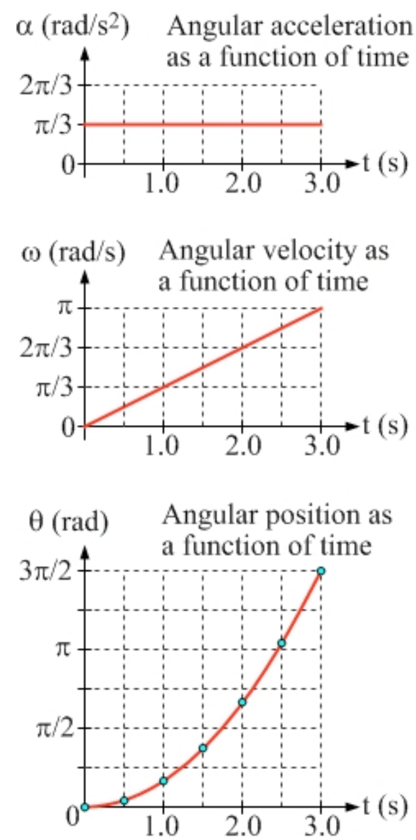

Figure 10.8: Graphs of the angular acceleration, angular velocity, and angular position for the object rotating with the turntable, all as a function of time.

Key idea: Plotting graphs of the angular acceleration, angular velocity, and angular position confirm the idea that rotational motion with constant angular acceleration is analogous to straight-line motion with constant acceleration, because the graphs in these two different situations have the same form. **Related End-of-Chapter Exercises: 40, 41.**

Essential Question 10.3: In Exploration 10.3B, we considered how to transform graphs for rotational motion into graphs for straight-line motion, but we did this with the special case of $r = 1.0$ m. What additional changes would be necessary if the radius r had a different value? Say, for example, that $r = 3.0$ m.