Answer to Essential Question 10.1: Let's re-arrange equation 10.1 to $\theta = s/r$. Thus, an arc length that is equal to the radius corresponds to an angle of 1.0 radian, which is about 57°. If the arc length and the radius have units of meters, the units cancel on the right side of the equation and we have units of radians on the left side. This violates the general rule that units have to match on two sides of an equation. We have two ways around this. One way is to treat the radian as dimensionless. Another way is to define the radius as having units of meters/radian.

10-2 Connecting Rotational Motion to Linear Motion

The angular variables we defined in Section 10-1 are vectors, so they have a direction. In which direction is the angular velocity of the disk shown in Figure 10.2? If we all observe the disk from the same perspective we can say that the direction is counterclockwise. In practice, we will generally use clockwise or counterclockwise to specify direction. In actuality, however, the direction is given by the **right-hand rule**. When you curl the fingers on your right hand in the direction of motion and stick out your thumb, your thumb points in the direction of the angular velocity. This is straight up out of the page for the disk in Figure 10.2.

EXPLORATION 10.2 – Connecting angular acceleration to acceleration

We can connect the magnitudes of the acceleration and angular acceleration in the same way that the distance traveled along an arc is connected to the angle ($s = r\theta$) and the speed is connected to the angular speed ($v = r\omega$). How?

Imagine yourself a distance *r* from the center of a rotating turntable, moving with the turntable. If the turntable has a constant angular velocity, you have no angular acceleration, but you have a centripetal acceleration, $\vec{a}_C = v^2/r$, directed toward the center of the turntable. The angular acceleration, α, cannot be connected to the centripetal acceleration by a factor of *r*, because $\alpha = 0$ in this case.

You have a non-zero angular acceleration if the turntable (and you) speeds up or slows down. If the turntable speeds up, the acceleration has two components (see Figure 10.4(a)), a centripetal acceleration $\vec{a}_C$ toward the center, and a component tangent to the circular path, which is called the tangential acceleration $\vec{a}_T$. If the turntable slows down, then the tangential acceleration reverses direction (see Figure 10.4(b)), as does the angular acceleration (because the angular velocity is decreasing instead of increasing). Thus, the magnitude of the tangential acceleration is directly related to the magnitude of the angular acceleration:

$a_T = r\alpha$ (Eq. 10.5: **Connecting tangential and angular accelerations**)

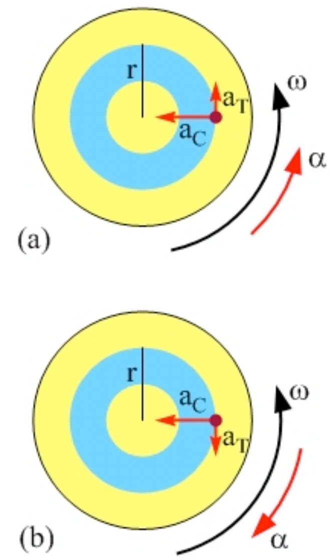


Figure 10.4: If you are rotating with a turntable as it speeds up (a) or slows down (b), your acceleration has two components, a centripetal component directed toward the center and a tangential component $\vec{a}_T$.

> **Key idea for angular acceleration**: The angular acceleration $\vec{\alpha}$ is directly related to the tangential acceleration $\vec{a}_T$ (the component of acceleration tangent to the circular path), and is not related to the centripetal acceleration $\vec{a}_C$. **Related End-of-Chapter Exercises: 44, 45.**

Equations for motion with constant angular acceleration

In Chapter 2, we considered one-dimensional motion with constant acceleration, and used three main equations to analyze motion. The analogous equations for rotational motion are summarized in Table 10.1. Note the parallels between the two sets of equations.

Straight-line motion equation		Analogous rotational motion equation	
$v = v_i + a\,t$	(Equation 2.9)	$\omega = \omega_i + \alpha\,t$	(Equation 10.6)
$x = x_i + v_i\,t + \frac{1}{2}a\,t^2$	(Equation 2.11)	$\theta = \theta_i + \omega_i\,t + \frac{1}{2}\alpha\,t^2$	(Equation 10.7)
$v^2 = v_i^2 + 2a\,\Delta x$	(Equation 2.12)	$\omega^2 = \omega_i^2 + 2\alpha\,\Delta\theta$	(Equation 10.8)

Table 10.1: Each kinematics equation has an analogous rotational-motion equation.

EXAMPLE 10.2 – Drawing a motion diagram for rotational motion

A turntable starts from rest, and has a counterclockwise angular acceleration of $(\pi/3)\text{ rad/s}^2$. Sketch a motion diagram for an object 1.0 m from the center that rotates with the turntable, plotting its position at 0.50 s intervals for the first 3.0 s.

SOLUTION

Let's use equation 10.7 to find the object's angular position at 0.50-second intervals. The object starts at the position shown by the red circle in Figure 10.5 – the horizontal line will be the origin. Take counterclockwise to be positive, and then set up a table (see Table 10.2) summarizing what we know. This is similar to what we did for one-dimensional motion.

Parameter	Value
Positive direction	Counterclockwise
Initial position	$\theta_i = 0$
Initial angular velocity	$\omega_i = 0$
Angular acceleration	$\alpha = +(\pi/3)\text{ rad/s}^2$

Table 10.2: Summarizing the initial information about the object.

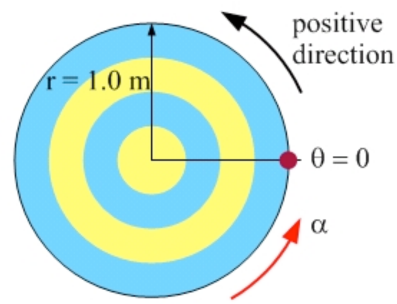


Figure 10.5: The initial situation for the rotating object.

Using the values from Table 10.2, Equation 10.7 simplifies to: $\theta = \theta_i + \omega_i\,t + \frac{1}{2}\alpha\,t^2 = 0 + 0 + \frac{1}{2}\left(\frac{\pi}{3}\text{rad/s}^2\right)t^2 = +\left(\frac{\pi}{6}\text{rad/s}^2\right)t^2$.

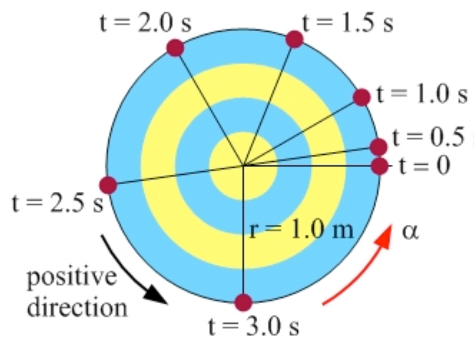
Substituting different times into this equation gives the angular position of the object at the times of interest, as summarized in Table 10.3.

Time (s)	0	0.50	1.00	1.50	2.00	2.50	3.00
Angular position (radians)	0	$+\pi/24$	$+\pi/6$	$+3\pi/8$	$+2\pi/3$	$+25\pi/24$	$+3\pi/2$
Angular position (°)	0	+7.5	+30	+67.5	+120	+187.5	+270

Table 10.3: The angular position of the object at 0.50-second intervals.

Using the information in Table 10.3, we can sketch a motion diagram for the object. The motion diagram is shown in Figure 10.6.

Figure 10.6: A motion diagram for an object moving with an accelerating turntable, showing the position at 0.5-second intervals.



Essential Question 10.2: If we repeated Example 10.2, for an object at a radius of 0.5 m from the center of the turntable, what would change in Table 10.3? Assume the object has an angular position of zero at $t = 0$.