Answer to Essential Question 10.11: Moving a supporting force farther from the center-of-gravity generally reduces that support force. However, the two supports together support the weight of the board. Thus, moving support B to the right decreases the magnitude of force B, but support A's upward force increases to compensate. The simplest example is when support B is at the far right of the board. In that case, symmetry tells us that the supports share the load equally, with each support exerting an upward force of 120 N on the board.

10-12 Further Investigations of Static Equilibrium

Center-of-gravity

Previously, we discussed the importance of locating forces precisely on a free-body diagram. For instance, the force of gravity must be attached to the center-of-gravity of the system. If the acceleration due to gravity has the same direction at all points in a system, we can define the *x*-coordinate of the system's center-of-gravity as:

$$X_{CG} = \frac{x_1 m_1 g_1 + x_2 m_2 g_2 + x_3 m_3 g_3 + \ldots}{m_1 g_1 + m_2 g_2 + m_3 g_3 + \ldots}$$ (Eq. 10.13: ***X*-coordinate of the center-of-gravity**)

A similar equation gives the *y*-coordinate of the center-of-gravity. If the acceleration due to gravity is the same everywhere, *g* cancels out of the equation, giving the center-of-mass equation we used in Chapter 6. The center-of-gravity differs from the center-of-mass, therefore, only when the acceleration due to gravity is different for different parts of the object or system.

EXAMPLE 10.12 – Tipping the board

Let's continue from where we left off in Example 10.11, involving the 240 N board on two supports. Now you climb on the board and, starting at the left end of the board, you slowly walk along the board toward the right end. Your weight is 480 N.

(a) Defining up to be the positive direction, plot two graphs, on the same set of axes, of the support forces as a function of your distance *d* from the left end of the board. Use this graph to determine the value of *d* when the board tips over.

(b) Where is the center-of-gravity, of the system consisting of you and the board, when the board begins to tip?

SOLUTION

(a) Again, we should draw a diagram to help us analyze the situation. Let's place you on the board at a distance *d* from the left end, and sketch a free-body diagram of the system. These diagrams are shown in Figure 10.29.

Let's define counterclockwise to be the positive direction for torques, and take torques about an axis perpendicular to the page that passes through the left end of the board. Choosing this axis eliminates, from the torque equation, the force exerted on the system by support A, and allows us to solve for the force exerted by support B. Let's use *M* to represent your mass and *m* to represent the mass of the board. Applying Newton's second law for rotation in this situation, $\sum \vec{\tau} = I\vec{\alpha} = 0$, gives:

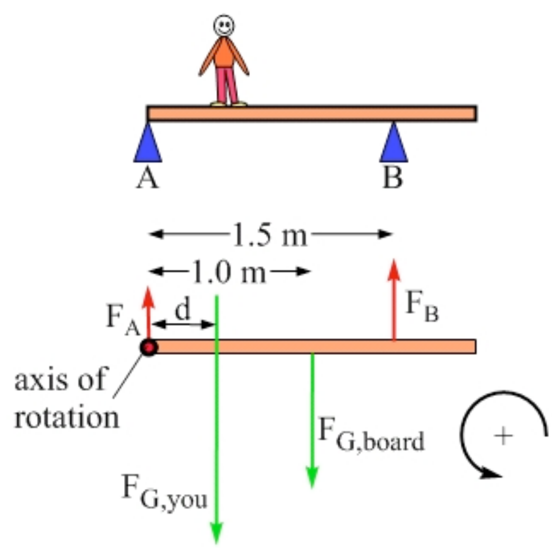

Figure 10.29: A diagram and free-body diagram of the system consisting of you and the board. You are a distance *d* from the left end of the board.

$$-d\,(Mg)\sin(90°)-(1.0\text{ m})(mg)\sin(90°)+(1.5\text{ m})\,F_B\sin(90°)=0\,.$$

Solving for the force exerted by support B gives:

$$F_B=\frac{2}{3}mg+\frac{2d}{3.0\text{ m}}Mg=160\text{ N}+320\,d\text{ N/m}\,.$$

We could follow a similar process to find an expression for the force exerted on the system by support A, taking torques about an axis through the board where support B is. In this case, however, it's probably easier to apply Newton's second law, $\sum\vec{F}=(M+m)\vec{a}=0$. Taking up to be positive gives: $+F_A-Mg-mg+F_B=0$.

Thus: $F_A=Mg+mg-F_B=480\text{ N}+240\text{ N}-160\text{ N}-320d\text{ N/m}=560\text{ N}-320d\text{ N/m}$.

Graphs of the two support forces, as a function of your position, are shown in Figure 10.30. Note that for values of $d > 1.75$ m, the force from support A must be negative (directed down) to maintain the system's equilibrium. A's force could be negative if the board was bolted to the support, and the support either had a significant mass or it was fastened firmly to the ground. In this case, however, the board simply rests on the support, so the support can only provide an upward force.

Thus, when $d = 1.75$ m, the board is on the verge of tipping, because the normal force between the board and support A goes to zero at that value of d. The board will tip if d exceeds 1.75 m. In this situation, then, Figure 10.30 shows the correct situation for $d \leq 1.75$ m.

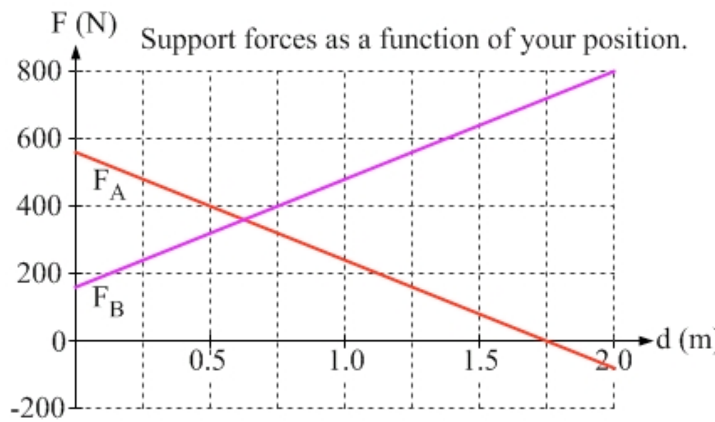

Figure 10.30: Graphs of the two support forces, as a function of your distance *d* from the left end of the board.

(b) What happens to the center-of-gravity of the system, which consists of you and the board, as you walk to the right? Because your weight is shifting right, the center-of-gravity of the system shifts right, also. The *y*-coordinate of the center-of-gravity has no bearing on whether the system tips, so let's simply determine the *x*-coordinate of the center-of-gravity when $d = 1.75$ m:

$$X_{CG}=\frac{x_{board}\,mg+x_{you}\,Mg}{mg+Mg}=\frac{(1.0\text{m})(240\text{ N})+(1.75\text{m})(480\text{ N})}{240\text{ N}+480\text{ N}}=1.5\text{ m}\,.$$

It is no coincidence that the position of the center-of-gravity corresponds to the location of support B. If the center-of-gravity of a system is between its supports, the system is stable. If the center-of-gravity moves out from the region bounded by the supports, the system tips over.

Related End-of-Chapter Exercises: 12, 34.

Essential Question 10.12: Return to the expressions we found for the support forces in part (a) of the Example 10.12. Add the two expressions. What is the significance of this result?