Answer to Essential Question 10.9: A familiar example is a figure skater who spins relatively slowly with her arms held out from her body, but then pulls her arms in and spins much faster.

10-10 Static Equilibrium

Let's first apply Newton's second law for rotation in a **static equilibrium** situation, in which an object remains at rest. Conditions for static equilibrium are given in section 10-9.

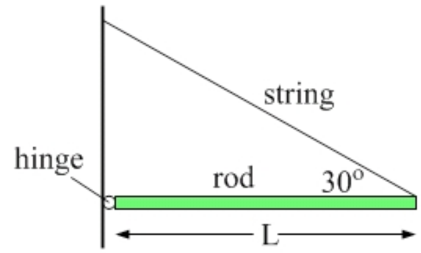

Figure 10.24: A diagram of the rod, connected to the wall by a hinge and held horizontal by a string tied to the end of the rod.

EXPLORATION 10.10 – A hinged rod.

Return to the hinged rod we looked at in Exploration 10.5. The rod's mass of 2.0 kg is uniformly distributed along its length L. The rod is attached to a wall by a hinge at one end. As shown in Figure 10.24, the angle between the rod and the string, which holds the rod in a horizontal position, is 30˚. Use g = 10 N/kg to simplify the calculations.

Step 1 – ***Sketch a free-body diagram of the rod.*** The free-body diagram is in Figure 10.25. Start by drawing the force of tension applied to the rod by the string, which goes away from the rod along the string. Where should we draw the force of gravity? Until now, all we had to do was to show the direction of a force correctly on a free-body diagram. Now that we're dealing with torques, it is also critical to locate the force accurately. The force of gravity should be drawn at the **center-of-gravity** of the rod, which is at the rod's geometrical center because the rod is uniform. For now, we can assume that the center-of-mass and the center-of-gravity are the same point. We'll distinguish between the two later in the chapter.

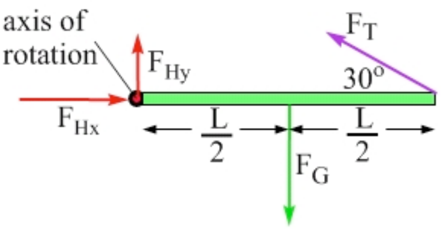

Figure 10.25: The free-body diagram of the rod.

What other forces act on the rod, in addition to gravity and tension? First, for the rod to remain in equilibrium, there must be a force directed right, to balance the component of the force of tension directed left. Second, because the hinge is in contact with the rod, the hinge very likely exerts a force on the rod. Generally, we draw this hinge force already split into components. The horizontal component of the hinge force, $\vec{F}_{Hx}$, is directed right to balance the horizontal component of the force of tension. The vertical component of the hinge force, $\vec{F}_{Hy}$, is shown directed up. This vertical component could, however, be directed down in some cases, or even be equal to zero. If you're not sure which direction a force is in, simply choose a direction. If the analysis gives a negative sign for a force, the force is opposite to the direction shown.

Step 2 – ***Apply Newton's second law twice, once for the horizontal direction and once for the vertical direction, to come up with two force equations for this situation.*** Figure 10.26 shows the x-y coordinate system, with positive x to the right and positive y up. The force of tension has been split into components parallel to the coordinate axes.

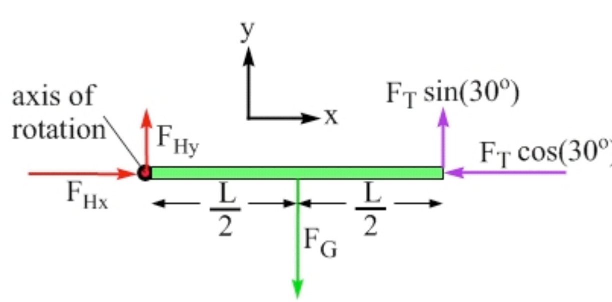

Figure 10.26: A free-body diagram showing the x-y coordinate system, and with all forces split into components.

Applying Newton's second law in the x-direction means:

1. Writing out Newton's second law: $\sum \vec{F}_x = m\,\vec{a}_x$.
2. Recognizing that the right-hand side equals zero, because the rod stays at rest.
3. Looking at the free-body diagram to evaluate the left-hand side of the equation: $+F_{H\,x} - F_T\cos(30°) = 0$.

Using a similar process for the y-direction, we start with $\sum \vec{F}_y = m\,\vec{a}_y$, and end up with:

$+F_{H\,y} - mg + F_T\sin(30°) = 0$

What can we solve for with these two force equations? We can't solve for anything! There are simply too many unknowns. If all we knew about were forces, we would be stuck.

Step 3 – ***Choose an appropriate axis to take torques about. Then, apply Newton's second law for rotation to write a torque equation to solve for the tension.*** What is an appropriate axis to use? Any axis can be used, but choosing an axis carefully can make a problem significantly easier to solve. ***The key is to choose an axis that one or more of the unknown forces pass through, because forces passing through an axis do not give any torque about that axis.*** In this case, we're trying to solve for the tension in the string, so we should pick an axis that eliminates the other unknown forces (the hinge forces), if possible. The most appropriate axis here is the axis perpendicular to the page that passes through the hinge. An axis through the hinge eliminates the two hinge forces, and the horizontal component of the force of tension, from the torque equation.

As with forces, we are free to choose a positive direction for torque. Let's use clockwise in this particular situation (although counterclockwise would be just as good). Applying Newton's second law for rotation means:

1. Writing out the equation: $\sum \vec{\tau} = I\vec{\alpha}$.
2. Recognizing that the right-hand side equals zero, because the rod stays at rest.
3. Looking at the free-body diagram to evaluate the left-hand side of the equation, and applying $\tau = r\,F\sin\theta$ to find the magnitude of the torque from each force. Recognizing that the torque due to the force of gravity is clockwise, while the torque due to the tension is counterclockwise, we get: $-L[F_T\sin(30°)]\sin(90°) + \frac{L}{2}(mg)\sin(90°) = 0$.

This equation demonstrates the power of using torque, because we can immediately solve for the tension. Note that the length of the rod, which is unknown, cancels out in the equation.

This gives: $F_T\sin(30°) = \frac{mg}{2}$.

Because $\sin(30°) = 0.5$, we find that $F_T = mg = (2.0\text{ kg})(10\text{ N/kg}) = 20\text{ N}$. Note that the fact that the force of tension has the same magnitude as the force of gravity in this case is highly coincidental, and happened only because the factor of two difference in the distances of these forces from the axis of rotation was exactly balanced by the factor of ½ we got from $\sin(30°)$.

Key ideas: By analyzing situations in terms of torque as well as force, we can solve problems that cannot be solved using force concepts alone. One of the keys to using torque is to choose an appropriate axis to take torques around. This is generally an axis that one or more of the unknown forces passes through. **Related End-of-Chapter Exercises: 8, 51 – 53.**

Essential Question 10.10: Return to Exploration 10.10, and solve for the x and y components of the force exerted on the rod by the hinge.