10-1 Rotational Kinematics

Kinematics is the study of how things move. Rotational kinematics is the study of how rotating objects move. Let's start by looking at various points on a rotating disk, such as a compact disc in a CD player.

EXPLORATION 10.1 - A rotating disk

Step 1 – *Mark a few points on a rotating disk and look at their instantaneous velocities as the disk rotates.* Let's assume the disk rotates counterclockwise at a constant rate. Even though the rotation rate is constant, we observe that each point on the disk has a different velocity. The instantaneous velocities of five different points are shown in Figure 10.1. Points at the same radius have equal speeds, but their velocities are different because the directions of the velocities are different. We also observe that the speed of a point is proportional to its distance from the center of the disk.
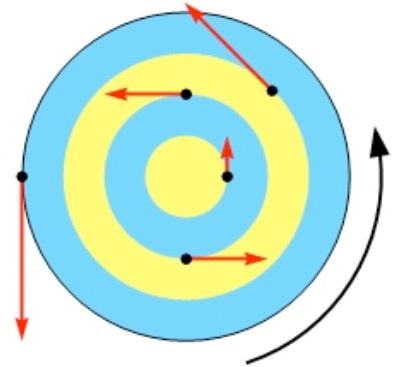
Step 2 – *Plot the paths followed by various points on the disk as the disk spins through 1/8th of a full rotation.* Figure 10.2 shows that each point travels on a circular arc, and the distance traveled by a particular point increases as the distance of that point from the center increases.

Figure 10.1: Instantaneous velocities, shown as red arrows, of various points on a disk rotating counterclockwise.

Step 3 – *What is the same for all the arcs shown in Figure 10.2?* One thing that is the same is the angle (45˚, in this case) the points move through, measured from the center of the disk. This leads to an interesting conclusion. Maybe the parameters we used (position, velocity, and acceleration) to study projectile and one-dimensional motion are not the most natural parameters to use when describing rotational motion. For instance, in a given time interval every point on the rotating disk has a unique displacement, yet each point has the same angular displacement.
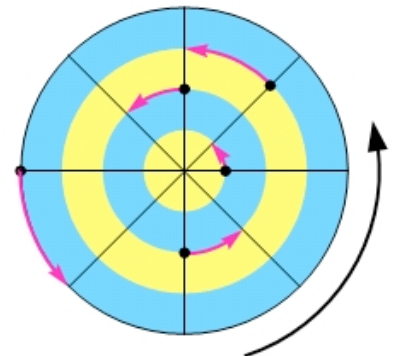
Step 4 – *Is there a connection between the distance traveled by a particular point in a given time interval (let's call this the arc length, s) and the corresponding angle, θ, of the arc the point moves along?*
Absolutely. The connection between arc length and angle is:

$$s = r\theta \qquad \text{(Equation 10.1: \textbf{Arc length})}$$

Figure 10.2: The circular arcs followed by five different points on the disk, as the disk moves through 1/8th of a full rotation.

where *r* is the radius of the arc (the distance from the point to the center). Note that the angle must be in units of radians in this equation, and that π radians $= 180˚$.

There is an equivalent relationship for speed. Each point on the disk has a unique velocity, but each point moves through the same angle in a given time interval. Thus, every point has the same angular velocity, a quantity we symbolize using the Greek letter omega, $\vec{\omega}$. Angular velocity is related to angular displacement $\Delta\vec{\theta}$ in the same way that velocity is related to displacement. $\vec{v} = \Delta\vec{r}/\Delta t$, so for the angular variables we have:

$$\vec{\omega} = \frac{\Delta\vec{\theta}}{\Delta t}. \qquad \text{(Equation 10.2: \textbf{Angular velocity})}$$

Step 5 – *What is the connection between the instantaneous velocity of a point on the rotating disk and the disk's angular velocity?* The instantaneous velocity of a point is called the tangential velocity, $\vec{v}_T$, because the direction of the velocity is always tangential to the circular path followed by the point. At a given instant in time, every point on the disk has the same angular speed ω (this is the magnitude of the angular velocity, $\vec{\omega}$). As we noted in Step 1, however, the speed of a particular point is proportional to r, its distance from the center. The connection between the tangential speed v_T, and the angular speed ω is:

$$v_T = r\omega.$$ (Eq. 10.3: **Connecting tangential speed to angular speed**)

Step 6 – *How do we describe motion when the rotating object is not rotating at a constant rate?* If the rotating disk spins with constant angular velocity we can fully describe the motion of any point on the disk using the parameters described above (and time, t). If the angular velocity is changing, however, such as if the disk is speeding up or slowing down, we need an additional parameter to describe motion. This is the angular acceleration $\vec{\alpha}$, the rotational equivalent of the acceleration, defined as:

$$\vec{\alpha} = \frac{\Delta\vec{\omega}}{\Delta t}.$$ (Equation 10.4: **Angular acceleration**)

Key ideas for rotational motion: To describe rotational motion, we use the rotational variables $\theta, \omega,$ and α. These are more natural variables to use, instead of the more familiar r, v, and a, because every point on a rotating object has the same angular velocity $\vec{\omega}$ and angular acceleration $\vec{\alpha}$, while each point has unique values of position, velocity, and acceleration.
Related End-of-Chapter Exercises: 1, 43.

Figure 10.3: In this time exposure image of a rotating Ferris wheel, note how the tracks left by the parts farther away from the center are longer than those made by parts closer to the center. Comparing this picture to the diagram of the rotating disk in Figure 10.2, where we see the length of the tracks increasing as the distance from the center increases, we can understand why parts farther from the center of the Ferris wheel leave longer tracks on the photograph.
Photo credit: Gisele Wright / iStockPhoto.

Essential Question 10.1: If you travel a distance of 1.0 m as you walk around a circle that has a radius of 1.0 m, through what angle have you walked? Comment on the units here.