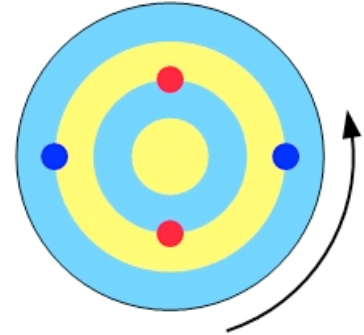


## End-of-Chapter Exercises

Exercises 1 – 12 are conceptual questions that are designed to see if you have understood the main concepts of the chapter.

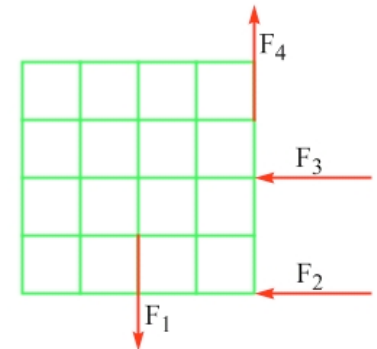
- As shown in the overhead view in Figure 10.31, four cylindrical objects (two red and two blue) are spinning with a turntable that is moving counterclockwise at a constant rate. The two red cylinders are the same distance from the center, and the two blue ones are also equally distant from the center, but farther from the center than the two red cylinders. Which cylinders have the same (a) speed? (b) velocity? (c) angular velocity? (d) acceleration? (e) angular acceleration?



**Figure 10.31:** An overhead view of a turntable that is spinning counterclockwise at a constant rate. Four cylinders are moving with the turntable, two red ones that are equally distant from the center, and two blue ones that are the same distance from the center as one another but farther out than the red ones. For Exercises 1 and 2.

- Return to the situation described in Exercise 2. (a) Draw a motion diagram for one of the red cylinders that corresponds to one complete rotation of the turntable. (b) Assuming the red cylinder is 2.0 m from the center of the turntable, construct a motion diagram that corresponds to the equivalent straight-line motion (as if you unrolled the motion diagram of the red cylinder you chose). Have we seen this kind of motion-diagram before? If so, what kind of motion did we classify it as?

- A square sheet of plywood is subjected to four forces of equal magnitude, as shown in Figure 10.32. Relative to an axis that is perpendicular to the page and passes through the top left corner of the sheet, in which direction is the torque due to (a)  $\vec{F}_1$ ; (b)  $\vec{F}_2$ ; (c)  $\vec{F}_3$ ; (d)  $\vec{F}_4$ ?



**Figure 10.32:** A square sheet of plywood subjected to four forces of equal magnitude, for Exercises 3 and 4.

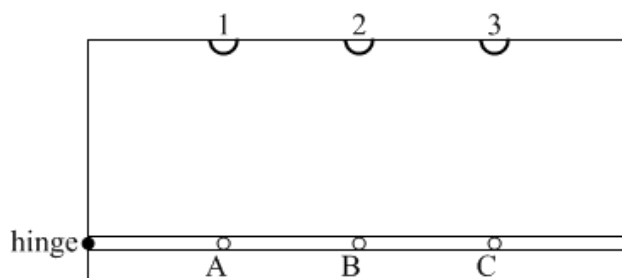
- Repeat Exercise 3, except this time use an axis that is perpendicular to the page and passes through the bottom left corner of the sheet.
- A hockey puck is initially at rest on a frictionless ice rink. Two horizontal forces of equal magnitude are then simultaneously applied to the puck. For rotation, consider a vertical axis through the center of the puck. (a) Is it possible to apply the two forces so the puck has no acceleration and no angular acceleration? If so, sketch an example. (b) Is it possible to apply the forces so the puck's center-of-mass has no acceleration but the puck has a non-zero angular acceleration? If so, sketch an example. (c) Is it possible to apply the forces so the puck's center-of-mass has a non-zero acceleration but the puck has no angular acceleration? If so, sketch an example.
- Many common household tools (hand tools, as opposed to power tools) enable us to make use of torque to make it easier to do something. A can opener is a good example of such a device. (a) Briefly describe how torque is involved in the operation of a human-powered can opener. (b) Name two other tools or devices you would find in a typical house that involve torque in their operation and briefly describe them.

7. Figure 10.33 shows a side view of a uniform rod of length  $L$  and mass  $M$  that is pinned at its left end by a frictionless hinge. The rod is held horizontal by means of a force  $F$  that is applied at a distance  $3L/4$  along the rod. The angle between the rod and this force is  $\theta$ . Fill in the two blanks in the following statement using either “increase”, “decrease”, or “stay the same.” As the angle  $\theta$  decreases, the torque associated with the force  $F$  must \_\_\_\_\_ while the magnitude of the force  $F$  must \_\_\_\_\_ so that the rod remains in equilibrium.



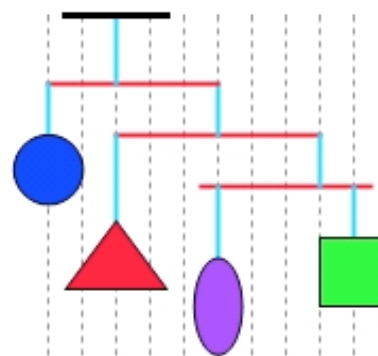
**Figure 10.33:** A side view of a rod that is hinged at its left end, and which is held in a horizontal position by an applied force  $F$ .

8. As shown in Figure 10.34, a rod, with a length of 80 cm and a mass of 6.0 kg, is attached to a wall by means of a hinge at the left end. The rod's mass is uniformly distributed along its length. A string will hold the rod in a horizontal position; the string can be tied to one of three points, lettered A-C, spaced at 20 cm intervals along the rod, starting with point A which is 20 cm from the left end of the rod. The other end of the string can be tied to one of three hooks, numbered 1-3, in the ceiling 30 cm above the rod. Hook 1 is directly above point A, hook 2 is directly above B, etc. For each case below, draw a line (and only one line) from one lettered point to one numbered hook representing the string you would use to achieve the desired objective. If you think it is impossible to achieve the objective, explain why. (a) How would you attach a string so the rod is held in a horizontal position with the hinge exerting no force at all on the rod? (b) How would you attach a string so the rod is held in a horizontal position while the force exerted on the rod by the hinge has no horizontal component, but has a non-zero vertical component directed straight up? (c) How would you attach a string so the rod is held in a horizontal position while the force exerted on the rod by the hinge has no vertical component, but has a non-zero horizontal component?



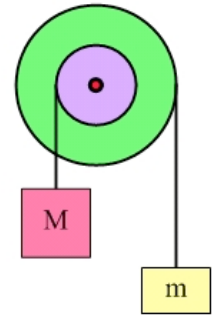
**Figure 10.34:** A hinged rod that you intend to hold horizontal by means of a string attached from one of the lettered points on the rod to one of the numbered hooks above the rod, for Exercise 8. This system represents a simple model of a broken arm you want to immobilize with a sling. The rod represents the lower arm, the hinge represents the elbow, and the string represents the sling.

9. You construct a mobile out of four objects, a sphere, a cube, a pyramid, and an ellipsoid. The mobile is in equilibrium in the configuration shown in Figure 10.35, where the vertical dashed lines are 20 cm apart. The mass of the strings (in blue) and rods (in red) can be neglected. If the pyramid has a mass of 400 g, what is the mass of each of the other objects?

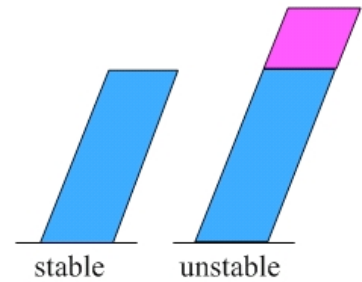


**Figure 10.35:** A mobile with four different objects, for Exercise 9.

10. Two cylinders have the same dimensions and mass, and the center-of-mass of each is at its geometric center. When you try to spin them, however, you notice that one cylinder is significantly more difficult to spin than the other. What is a good physical explanation for this? Assume you're trying to spin them about an axis through the center of the cylinder, perpendicular to the length of the cylinder, in each case.
11. A pulley consists of a small uniform disk of radius  $R/2$  mounted on a larger uniform disk of radius  $R$ . The pulley can rotate without friction about an axis through its center. As shown in Figure 10.36, a block with mass  $m$  hangs down from the larger disk while a block of mass  $M$  hangs down from the smaller disk. If the system remains in equilibrium, what is  $M$  in terms of  $m$ ?
12. A particular type of leaning tower toy, as shown in Figure 10.37, remains upright until an extra piece is added to its top, at which point the tower falls over. Explain why this is.



**Figure 10.36:** A dual-radius pulley system remains in equilibrium with two blocks hanging from it. For Exercise 11.



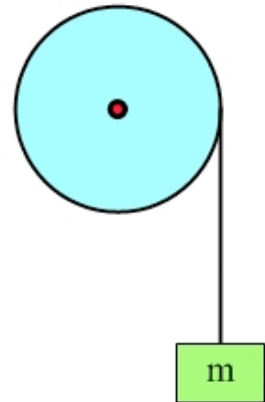
**Figure 10.37:** This leaning-tower toy tips over when the extra piece is added at the top. For Exercise 12.

**Exercises 13 – 21 are designed to give you practice in solving a typical rotational kinematics problem.** For each exercise, start with the following parts: (a) Draw a diagram of the situation. (b) Choose an origin to measure displacements from and mark that on the diagram. (c) Choose a positive direction and indicate that with an arrow on the diagram. (d) Create a table summarizing everything you know, as well as the unknowns you want to solve for. Try to solve all exercises using a similar systematic approach. Compare your approach to those you used for Exercises 33 – 42 in Chapter 2.

13. While repairing your bicycle, you have your bicycle upside down so the front wheel is free to spin. You grab the front wheel by the edge and smoothly accelerate it from rest, giving it an angular acceleration of  $5.0 \text{ rad/s}^2$  clockwise. You let go after the wheel has moved through one-quarter of a revolution. Your goals in this exercise are to determine the wheel's angular velocity at the instant you let go and the time it took to reach that angular velocity. Parts (a) – (d) as described above. (e) Which equation(s) will you use to determine the wheel's final angular velocity? (f) Find that angular velocity. (g) Which equation(s) will you use to determine the time the wheel was accelerating? (h) Solve for that time.
14. You release a ball from rest at the top of a ramp, and it experiences a constant angular acceleration of  $1.2 \text{ rad/s}^2$ . At the bottom of the ramp, the ball is rotating at 4.0 revolutions per second. The goal here is to determine how long it took the ball to reach that speed. Parts (a) – (d) as above. (e) Which equation(s) will you use to determine the time it takes to reach 4.0 rev/s? (f) What is that time?
15. You spin a disk, giving it an initial angular velocity of  $2.4 \text{ rad/s}$  clockwise. The disk has an angular acceleration of  $1.2 \text{ rad/s}^2$  counterclockwise. Your goal in this exercise is to solve for the maximum angular displacement of the disk from its initial position before reversing direction. Parts (a) – (d) as described above. (e) Which equation(s) will you use to determine the maximum angular displacement of the disk? (f) Solve for that angular displacement.

16. In this exercise, you will analyze the method you used in Exercise 15, so all these questions pertain to what you did to solve Exercise 15. (a) Is there only one correct choice for the origin? Why did you make the choice you made? (b) Is there only one correct choice for the positive direction? Would your answer to (f) above change if you chose the opposite direction to be positive? (c) Find an alternative method to determine the maximum angular displacement, and show that it gives the same answer as the method you used.
17. A cylinder is rolling down a ramp. When it passes a particular point, you determine that it is traveling at an angular speed of  $30 \text{ rad/s}$ , and in the next  $2.0$  seconds it experiences an angular displacement of  $80$  radians. The goal of this exercise is to determine the cylinder's angular acceleration, which we will assume to be constant. Parts (a) – (d) as described above. (e) Which equation(s) will you use to determine the angular acceleration? (f) What is the angular acceleration?
18. Repeat parts (e) and (f) of the previous exercise, but do not use the equation(s) you used in the previous exercise.

19. A pulley with a radius of  $0.20 \text{ m}$  is mounted so its axis is horizontal. A block hangs down from a string wrapped around the pulley, as shown in Figure 10.38. You give the pulley an initial angular velocity of  $0.50 \text{ rad/s}$  directed counterclockwise. The block takes a total of  $6.00 \text{ s}$  to return to the level it was at when you released it. Assuming the acceleration is constant through the entire motion, the goal of the exercise is to determine the maximum distance the block rises above its initial point. Parts (a) – (d) as described above. (e) Which equation(s) will you use to determine the maximum distance the block rose above its initial point? (f) What is that maximum distance?

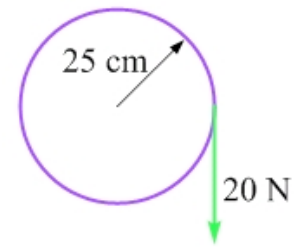


**Figure 10.38:** A block hanging down from a pulley, for Exercise 19.

20. An electric drill accelerates a drill bit, which has a radius of  $3.0 \text{ mm}$ , from rest to a maximum angular speed of  $250 \text{ rpm}$  (revolutions per minute) in  $2.2$  seconds. The goal of this exercise is to determine the drill bit's angular acceleration, assuming it to be constant. Parts (a) – (d) as described above. (e) What is the bit's angular acceleration?
21. With a quick flick of her wrist, an Ultimate Frisbee player can give a Frisbee an angular velocity of  $8.0$  revolutions per second. Assuming the player accelerates the Frisbee from rest through an angle of  $75^\circ$ , the goal of this exercise is to determine the Frisbee's angular acceleration and the time over which this acceleration occurs. Parts (a) – (d) as described above. (e) What is the Frisbee's angular acceleration? (f) What is the time over which the acceleration occurs?

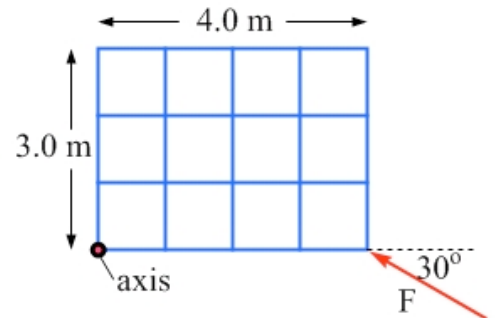
Exercises 22 – 26 involve calculating torque in various situations.

22. As shown in Figure 10.39, a disk mounted on an axle through its center is subjected to a 20 N force. The disk has a radius of 25 cm. What is the magnitude and direction of the torque associated with this force, measured with respect to an axis that is perpendicular to the page and
- passes through the center of the disk?
  - passes through the point 50 cm above the center of the disk (i.e., move the axis up the page)?
  - 50 cm to the right of the center of the disk.



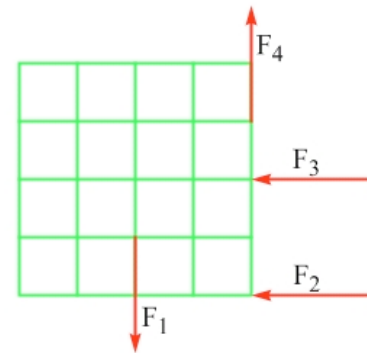
**Figure 10.39:** A disk of radius 25 cm that is subjected to a 20 N force, for Exercise 22.

23. A box measuring 3.0 m high by 4.0 m wide is subjected to a 10 N force, as shown in Figure 10.40. Consider an axis that is perpendicular to the page and which passes through the bottom left corner of the box.
- Follow the procedures outlined in Exploration 10.6 to first determine the direction of the torque due to this force. Now determine the magnitude of the torque due to this force by
  - applying Equation 10.9 directly;
  - breaking the force into horizontal and vertical components before applying Equation 10.9;
  - using the lever arm method.



**Figure 10.40:** A box subjected to a 10 N force, for Exercise 23.

24. The plywood sheet shown in Figure 10.41 measures 2.0 m  $\times$  2.0 m, and each of the four forces the sheet is subjected to has a magnitude of 8.0 N. Relative to an axis that is perpendicular to the page and passes through the top left corner of the sheet, determine the magnitude of the torque due to
- $\vec{F}_1$ ;
  - $\vec{F}_2$ ;
  - $\vec{F}_3$ ;
  - $\vec{F}_4$ .
- (e) Find the magnitude and direction of the net torque, due to all four forces, about that axis.
25. Repeat Exercise 24, except this time use an axis that is perpendicular to the page and passes through the bottom left corner of the sheet.

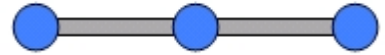


**Figure 10.41:** A square sheet of plywood subjected to four forces of equal magnitude, for Exercises 24 – 26.

26. Consider again the plywood sheet shown in Figure 10.41. Is there an axis that is perpendicular to the page about which the four forces give a net torque of zero? If so, where would such an axis be located? If there is at least one such axis, is there only one, or are there more than one? Explain.

**Exercises 27 – 32 address issues associated with rotational inertia.**

27. As shown in Figure 10.42, three identical balls, each with a mass  $M = 1.0$  kg, are equally spaced along a rod of negligible mass. The distance between neighboring balls is 3.0 m, and you can assume that the radius of each ball is considerably less than the distance between them. If this system is spun about an axis that is perpendicular to the page, determine the system's total rotational inertia if the axis (a) passes through the center of the middle ball; (b) passes through the center of the ball at the left end.



**Figure 10.42:** A system of three balls, for Exercise 27.

28. A solid sphere has a mass of  $M = 8.0$  kg and a radius of  $R = 20$  cm. Determine the sphere's rotational inertia about an axis (a) passing through the center of the sphere; (b) tangent to the outer surface of the sphere.
29. Two balls of negligible radius are connected by a rod with a length of 1.2 m and a negligible mass. One ball has a mass  $M$ , while the other has a mass of  $2M$ . (a) If you spin this system about an axis that is perpendicular to the rod, where should you place the axis to minimize the system's rotational inertia? (b) If  $M = 1.0$  kg, what is this minimum rotational inertia?
30. Repeat Exercise 29, except now the balls are joined by a 1.2-meter uniform rod with a mass of  $3M$ .
31. Four balls of equal mass  $M$  are placed so that there is one ball at each corner of a square measuring  $d \times d$ . The balls are joined by rods of negligible mass that run along the sides of the square. Assume the radius of each ball is small compared to  $d$ . What is the rotational inertia of the system about an axis that is perpendicular to the plane of the square and passes through (a) the center of the square? (b) one of the corners of the square? (c) a distance  $d$  from the center of the square, in any direction.
32. The rotational inertia of a uniform rod of length  $L$  and mass  $M$ , about an axis through the end of the rod and perpendicular to the rod, is  $I = ML^2 / 3$ . Use this expression, and the parallel-axis theorem (Equation 10.10), to show that the rotational inertia of the rod about a parallel axis through the center of the rod is  $I = ML^2 / 12$ .

**Exercises 33 – 38 are designed to give you practice in solving a typical static equilibrium problem.** For each of these exercises begin with the following: (a) Draw a diagram of the situation. (b) Draw a free-body diagram to show each of the forces acting on the object.

33. A board with a weight of 40 N and a length of 2.0 m is placed horizontally on a flat roof with 75 cm of the board hanging over the edge of the roof. The goal of this exercise is to determine the magnitude and direction of the normal force exerted on the board by the roof, and the exact location the normal force can be considered to be applied. Parts (a) and (b) as described above. (c) Apply Newton's second law to your free-body diagram, and solve for the magnitude and direction of the normal force exerted on the board by the roof. (d) Choose an axis to take torques about. Why did you select the axis you did? (e) Apply Newton's second law for rotation to determine the location the normal force can be considered to act on the board.

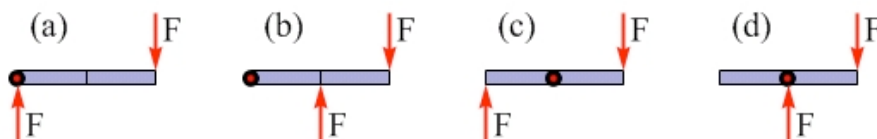


34. Return to the situation described in Exercise 33, except now we will add a 20 N bucket of nails to the end of the board that is hanging out over the edge of the roof. Repeat parts (a) – (e). (f) What is the maximum weight that could be placed on the end of the board without the board tipping over? (g) Where would the normal force act in that situation?
35. A particular door consists of a uniform piece of wood, with a weight of 20 N, measuring 2.0 m high by 1.0 m wide. The door is mounted on two hinges, one 20 cm down from the top of the door and the other 20 cm up from the bottom. The door also has an ornate handle, with a weight of 10 N, located halfway down the door and 10 cm from the edge of the door farthest from the hinges. The goal of this exercise is to determine the horizontal components of each hinge force. Parts (a) – (b) as described above. (c) Apply Newton's second law in the horizontal direction to obtain a relationship between the horizontal components of the hinge forces. (d) Now, choose an axis to take torques about so you can solve for the horizontal component of the force exerted on the door by the bottom hinge. Explain why you chose the axis you did. (e) Apply Newton's second law for rotation and solve for the horizontal component of the force exerted on the door by the bottom hinge. (f) Solve for the horizontal component of the force exerted on the door by the upper hinge.
36. Consider the following design for a one-person seesaw. The seesaw consists of a uniform board with a length of 5.0 m and a mass of 40 kg balanced on a support that is 2.0 m from one end. Julie, with a mass of 16 kg, sits on the board so that the system is in balance with the board horizontal. The goal is to determine how far from the support Julie is. Parts (a) – (b) as described above. (c) Which side of the board is Julie on, the side with 2.0 m or the side with 3.0 m of the board extending beyond the support? (d) Choose an axis to take torques about. Justify your choice of axis. (e) Choose a positive direction for rotation and apply Newton's second law for rotation to find Julie's position.
37. A ladder with a length of 5.0 m and a weight of 600 N is placed so its base is on the ground 4.0 m from a vertical frictionless wall, and its tip rests 3.0 m up the wall. The ladder remains in this position only because of the static friction force between the ladder and the ground. The goal of this exercise is to determine the magnitude of the normal force exerted by the wall on the ladder, and the minimum possible value of the coefficient of static friction for the ladder-ground interaction. Assume the mass of the ladder is uniformly distributed. Parts (a) – (b) as described above. (c) Apply Newton's second law twice, once for the horizontal forces and once for the vertical forces, to find relationships between the various forces applied to the ladder. (d) Choose an axis to take torques about so that you can solve for the normal force exerted by the wall on the ladder. Justify why you chose the axis you did. (e) Choose a positive direction for rotation and apply Newton's second law for rotation to find the normal force exerted by the wall on the ladder. (f) Solve for the minimum possible coefficient of static friction so the ladder remains in equilibrium.
38. Repeat Exercise 37, with the addition of you, with a weight of 500 N, standing on the ladder so that you are a horizontal distance of 1.0 m from the wall.

### General Problems and Conceptual Questions

39. You drop a ball from rest from the top of a tall building. Let's assume the ball has an acceleration of  $10 \text{ m/s}^2$  directed straight down. At the same time you drop the ball, you flick a switch that starts a motor, giving a disk that was initially at rest an angular acceleration of  $10 \text{ rad/s}^2$  directed clockwise. Write a paragraph or two comparing and contrasting these two motions.

40. Return to the situation described in Exercise 39. (a) For the first four seconds of the motion, plot graphs of the ball's acceleration, velocity, and position as a function of time, taking down to be positive. (b) Over the same time period, plot graphs of the disk's angular acceleration, angular velocity, and angular position as a function of time, taking clockwise to be positive. (c) Comment on the similarities between the two sets of graphs.
41. Consider again the situation described in Exercise 39. The disk rotates about an axis through its center. It turns out that all points on the disk at a particular distance from the disk have a speed that matches the speed of the ball at all times (at least until the ball hits the ground!). What is this distance?
42. In the "old days", long before CD's and MP3's, people listened to music using vinyl records. Long-playing vinyl records spin at a constant rate of  $33\frac{1}{3}$  rpm. The music is encoded into a continuous spiral track on the record that starts at a radius of about 30 cm from the center and ends at a radius of about 10 cm from the center. If a record plays for 24 minutes, estimate how far apart the tracks are on the record.
43. Let's say that, during your last summer vacation, you drove your car across the United States from Boston to Seattle, staying on interstate I-90 the entire time. Estimate how many revolutions each tire made during this trip.
44. You are driving your car at a constant speed of 20 m/s around a highway exit ramp that is in the form of a circular arc of radius 100 m. What is the magnitude of your (a) angular velocity? (b) centripetal acceleration? (c) tangential acceleration? (d) angular acceleration?
45. Repeat Exercise 44, except now your speed is decreasing. At the instant we are interested in, your speed is 20 m/s, and you are planning to come to a complete stop at a red light in 5.0 s. Assume your acceleration is constant.
46. The rotational inertia of a uniform disk of mass  $M$  and radius  $R$  is found to be  $MR^2$  about an axis perpendicular to the plane of the disk. How far is this axis from the center of the disk?
47. Archimedes once made a famous statement about a lever, which relies very much on the principle of torque, using words to the effect of "Give me a lever long enough, and a fulcrum on which to place it, and I shall move the world." Briefly explain what Archimedes was talking about.
48. Figure 10.43 shows four different cases involving a uniform rod of length  $L$  and mass  $M$  that is subjected to two forces of equal magnitude. The rod is free to rotate about an axis that either passes through one end of the rod, as in (a) and (b), or passes through the middle of the rod, as in (c) and (d). The axis is marked by the red and black circle, and is perpendicular to the page in each case. This is an overhead view, and we can neglect any effect of the force of gravity acting on the rod. Rank these four situations based on the magnitude of the net torque about the axis, from largest to smallest.



**Figure 10.43:** Four situations involving a uniform rod, which can rotate about an axis, being subjected to two forces of equal magnitude. For Exercises 48 and 49.



49. Return to the situation described in Exercise 48 and shown in Figure 10.43. If the rod has a length of 1.0 m, a mass of 3.0 kg, and each force has a magnitude of 5.0 N, determine the magnitude and direction of the net torque on the rod, relative to the axis in (a) Case (a); (b) Case (b); (c) Case (c); (d) Case (d).

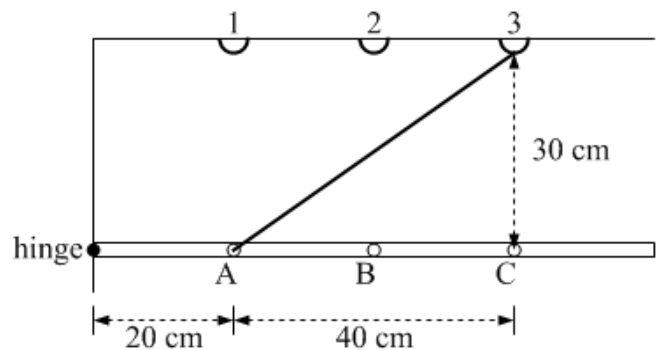
50. Figure 10.44 shows a side view of a uniform rod of length  $L$  and mass  $M$  that is pinned at its left end by a frictionless hinge. The rod is held horizontal by means of a force  $F$  that is applied at a distance  $3L/4$  along the rod.

Determine the angle  $\theta$  between the rod and the force  $F$ , such that the magnitude of  $F$  is exactly equal to the magnitude of the force of gravity acting on the rod.



**Figure 10.44:** A side view of a rod that is hinged at its left end, and which is held in a horizontal position by an applied force  $F$ . For Exercise 50.

51. A rod, with a length of 80 cm and a mass of 6.0 kg, is attached to a wall by means of a hinge at the left end. The rod's mass is uniformly distributed along its length. A string will hold the rod in a horizontal position; the string can be tied to one of three points, lettered A-C, spaced at 20 cm intervals along the rod, starting with point A which is 20 cm from the left end of the rod. The other end of the string can be tied to one of three hooks, numbered 1-3, in the ceiling 30 cm above the rod. Hook 1 is directly above point A, hook 2 is directly above B, etc. Use  $g = 10 \text{ m/s}^2$ . As shown in Figure 10.44, a string is attached from point A to hook 3. Remember that point B is 40 cm from the hinge. (a) Calculate the tension in the string. (b) Determine the magnitude and direction of the horizontal component of the hinge force. (c) Determine the magnitude and direction of the vertical component of the hinge force.

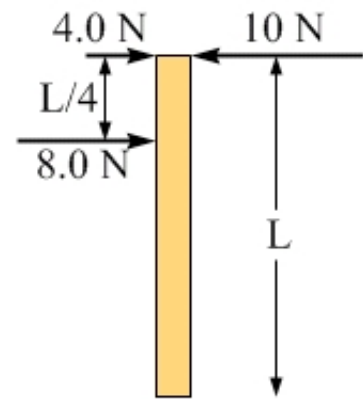


**Figure 10.45:** A diagram of a hinged rod, held horizontal by a string. Point B, in the middle of the uniform rod, is 40 cm from the hinge. For Exercises 51 – 53.

52. Repeat Exercise 51, with the string holding the rod horizontal attached from point A to hook 2 instead.
53. Repeat Exercise 51, with the string holding the rod horizontal attached from point C to hook 2 instead.
54. It is often useful to treat the lower arm as a uniform rod of length  $L$  and mass  $M$  that can rotate about the elbow. Let's say you are holding your arm so your upper arm is vertical (with your elbow below your shoulder), with a  $90^\circ$  bend at the elbow so the lower arm is horizontal. In this position we can say that three forces act on your lower arm: the force of gravity ( $Mg$ ), the force exerted by the biceps, and the force exerted at the elbow joint by the humerus (the bone in the upper arm). Let's say the biceps muscle is attached to the lower arm at a distance of  $L/10$  from the elbow, moving away from the elbow toward the hand. (a) Compare the force of gravity with the biceps force. Which has the larger magnitude? Briefly justify your answer. (b) Compare the force from the biceps with the force from the humerus. Which has the larger magnitude? Briefly justify your answer.

55. Return to the situation described in Exercise 54. If you now hold a 20 N object in your hand, at a distance  $L$  from the elbow joint, and your arm remains in the position described, the force from the biceps increases. (a) By how much does the force from the biceps increase? (b) Does the fact that you are holding a 20 N object in your hand change the force applied to your lower arm by the humerus at the elbow joint? If so, state both the magnitude and the direction of the change.
56. A uniform rod of mass  $M$  and length 1.0 m is attached to a wall by a hinge at one end. The rod is maintained in a horizontal position by a vertical string that can be attached to the rod at any point between 20 cm and 100 cm from the hinge. (a) Defining  $d$  to be the distance from the hinge to where the string is attached, plot a graph of the magnitude of the torque exerted on the rod by the string as a function of  $d$ , for  $20\text{ cm} \leq d \leq 100\text{ cm}$ . (b) Over the same range of  $d$  values, plot a graph of the magnitude of the tension in the string as a function of  $d$ .

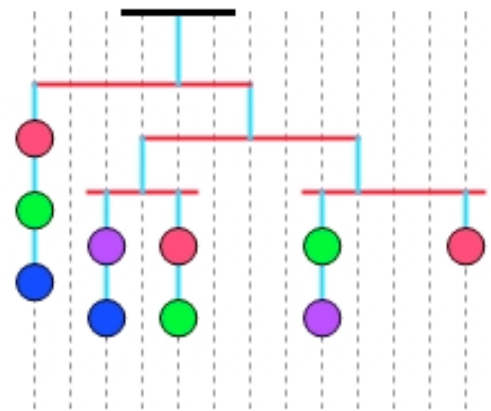
57. Figure 10.46 shows an overhead view of a piece of wood, with a mass of 2.0 kg, that is on a slippery ice rink. Three horizontal forces are shown on the rod, and there is a fourth force of unknown magnitude, direction, and location that is not shown. Determine the magnitude, direction, and location of the mystery force if the piece of wood remains at rest.



58. Consider again the situation described in the previous exercise and shown in Figure 10.46. Is it possible to apply a fourth force so the piece of wood does not spin but accelerates at  $3.0\text{ m/s}^2$  to the right? Justify your answer.

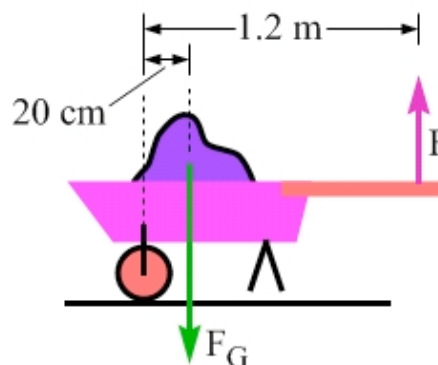
**Figure 10.46:** An overhead view of a piece of wood on a slippery ice rink. Four forces are applied to the wood but only three are shown. For Exercises 57 and 58.

59. Consider the mobile shown in Figure 10.47, in which all the balls have equal mass and in which the weight of the vertical strings and horizontal rods can be neglected. The vertical dashed lines in the figure are 10 cm apart. (a) Is this mobile in equilibrium in the configuration shown? How do you know? (b) If you add one additional ball to the configuration shown in the diagram can you get the mobile to be in equilibrium? If not, explain why not. If so, explain where you would place the additional ball. (c) If you have concluded that the mobile is not an equilibrium as shown, and that adding one additional ball will not achieve equilibrium, what is the minimum number of balls you can add to the system to achieve equilibrium, and where would you put them?



**Figure 10.47:** A mobile consisting of several balls of equal mass, for Exercise 59.

60. You are using a wheelbarrow to move a heavy rock, as shown in Figure 10.48. The diagram shows the location of the upward force you exert and the location of the force of gravity acting on the rock – wheelbarrow system. (a) Assuming the wheelbarrow is in equilibrium, how does the magnitude of the force you exert compare to the magnitude of the force of gravity acting on the system? (b) If the force of gravity has a magnitude of 420 N, solve for the magnitude of the force you exert on the wheelbarrow, and the magnitude and direction of any other force or forces necessary to maintain equilibrium.



**Figure 10.48:** A side view of a wheelbarrow you are using to move a heavy rock, for Exercise 60.

61. Consider again the one-person seesaw described in Exercise 36. Would your answer for where Julie must sit to maintain equilibrium change if the system was on the Moon, as opposed to the Earth? Justify your answer.
62. Two of your friends, Latisha and Jorge, are carrying on a conversation about a physics problem. Comment on each of their statements, and state the answer to the problem the two of them are working on.

*Latisha: In this problem, we have a uniform rod with a weight of 12 newtons, and it is supported at one end by a hinge. The question is, what is the smallest force that we can apply to keep the rod horizontal? Don't we need to apply a 12 newton force, at least, to hold it up?*

*Jorge: I think the idea is that the hinge can help support some of the weight, so, if we hold it in the right spot, we can apply a force that is less than 12 newtons.*

*Latisha: What if we start in the middle? If we hold it in the middle, then the rod is perfectly balanced, and we don't need the hinge at all. In that case, then we'd just need to apply a 12 newton force up to balance the 12 newton force of gravity, right?*

*Jorge: I think so. So, then, if we want to apply less force, should we move our force toward the hinge or away from the hinge? How do we figure that out if we don't know what the hinge force is?*

*Latisha: I guess this is why we're learning about torques. If we take torques about the hinge, then I think the hinge force cancels out – the distance is zero, for the torque from the hinge force. Then, if we apply our force farther from the hinge, can't we apply less force? But, how do we know how much less? We don't even know the length of the rod!*