Answer to Essential Question 9.7: The increase in pressure with depth is proportional to the product of the density multiplied by the vertical distance. Because the density of the water is on the order of 1000 times larger than that of air, we can neglect this effect for the air.

9-8 Fluid Dynamics

Let's turn now from analyzing fluids at rest to analyzing fluids in motion. The study of flowing fluids is known as fluid dynamics. Fluid dynamics can be rather complex, so we will make some simplifying assumptions. These include:

1. The flow is steady – flow patterns are maintained without turbulence.
2. The fluid is incompressible – its density is constant.
3. The fluid is non-viscous – there is no resistance to the flow.
4. The flow is irrotational – there are no swirls or eddies.

Under these assumptions we get what is called streamline flow, indicated by the blue streamlines in the pipe shown in Figure 9.22.

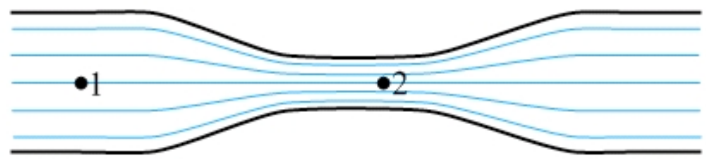

Figure 9.22: Streamline flow through a pipe.

Continuity

There are two main equations we will apply to analyze flowing fluids. The first of these is called the continuity equation, which comes from the fact that when an incompressible fluid flows through a tube of varying cross-section, the rate at which mass flows past any point in the tube is constant. If the flow rate varied, fluid would build up at points where the flow rate is low.

The mass flow rate is the total mass flowing past a point in a given time interval divided by that time interval. At a point where the flow is in the x-direction with a speed v and the tube has a cross-sectional area A, the mass flow rate is given by:

$$\text{mass flow rate} = \frac{\Delta m}{\Delta t} = \frac{\rho\ \Delta V}{\Delta t} = \frac{\rho\ A\, \Delta x}{\Delta t} = \rho\, A v\,.$$

Consider the streamline flow pattern in Figure 9.22. The mass flow rate is the same at two different points, 1 and 2, in the tube, so:

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2\,.$$

One of our assumptions is that the density is the same at all points, so we can reduce the preceding equation to:

$$A_1 v_1 = A_2 v_2\,.$$ (Equation 9.8: **The Continuity Equation**)

The main implication of the continuity equation is that the speed of the fluid increases as the cross-sectional area of the tube decreases, and vice versa. The streamlines in Figure 9.22 show the change in speed that corresponds to a change in area. Where the streamlines are farther apart, such as at point 1, the flow speed is less. Where the streamlines are close together, such as at point 2, the speed is higher.

The second equation we will apply to flowing fluids is an energy conservation equation, transformed to be particularly useful for fluids. Let's start by writing out our energy conservation equation from chapter 6:

$$U_1 + K_1 + W_{nc} = U_2 + K_2 .$$

The potential energy we're talking about here is gravitational potential energy, in the form $U = m\,g\,y$, and we can write the kinetic energy as $K = (1/2)mv^2$. The energy equation can thus be written as:

$$mgy_1 + \frac{1}{2}mv_1^2 + W_{nc} = mgy_2 + \frac{1}{2}mv_2^2 .$$

Let's apply this equation to a fluid flowing through a pipe. Figure 9.23 shows the two points we are considering, and two cylindrical regions of fluid are highlighted, one at each point. The cylindrical regions have equal volumes.

In the case of a flowing fluid, the work done by non-conservative forces is related to forces that arise because of pressure differences. We can write the W_{nc} term as:

$$W_{nc} = F_{nc}\,\Delta x = F_1\,\Delta x_1 - F_2\,\Delta x_2 = P_1 A_1\,\Delta x_1 - P_2 A_2\,\Delta x_2 .$$

Substituting this expression for W_{nc} into the energy conservation relationship gives:

$$mgy_1 + \frac{1}{2}mv_1^2 + P_1 A_1 \Delta x_1 = mgy_2 + \frac{1}{2}mv_2^2 + P_2 A_1 \Delta x_1 .$$

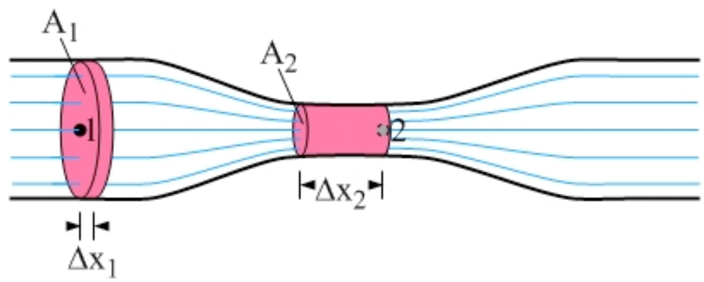

Figure 9.23: The two cylindrical regions, one at point 1 and one at point 2, have the same volume.

The *m* here represents the mass of the fluid in one of the cylindrical regions in Figure 9.23 (the cylindrical regions have equal masses because of their equal volumes).

Let's simplify the equation by dividing both sides by the volume *V* of one of the cylindrical regions ($V = A_1\,\Delta x_1 = A_2\,\Delta x_2$). Because mass/volume = density, we get:

$$\rho g y_1 + \frac{1}{2}\rho v_1^2 + P_1 = \rho g y_2 + \frac{1}{2}\rho v_2^2 + P_2 .$$ (Equation 9.9: **Bernoulli's Equation**)

Bernoulli's equation represents conservation of energy applied to fluids, although each term has units of energy density (energy per unit volume).

Related End-of-Chapter Exercises: 31, 56.

Essential Question 9.8: Consider a special case of Bernoulli's equation, when the fluid is at rest. Which of the equations we have examined previously in this chapter is equivalent to Bernoulli's equation with the two terms involving speed set to zero?