Answer to Essential Question 9.3: The second fluid has a density that is half that of water. We can see that because a particular volume of water has a mass that is twice as much as the mass of an equal volume of the second fluid.

9-4 Solving Buoyancy Problems

Archimedes was a Greek scientist who, legend has it, discovered the concept while taking a bath, whereupon he leapt out and ran naked through the streets shouting "Eureka!" Archimedes was thinking about this because the king at the time wanted Archimedes to come up with some way to make sure that the king's crown was made out of solid gold, and was not gold mixed with silver. Archimedes realized that he could use his principle to determine the density of the crown, and he could then compare it to the known density of gold.

Using Equation 9.3, we can now explain the results of the block-and-two-fluid experiment above. The differences we observe between when we place the blocks in water and when we place them in the second fluid can all be explained in terms of the difference between the density of water and the density of the second fluid. In fact, to explain the results of Exploration 9.2 the second fluid must have half the density of water. The 10-N block, for instance, floats in both fluids and therefore the buoyant force is the same in both cases, exactly equal-and-opposite to the 10 N force of gravity acting on the block. Because the density of the second fluid is half the density of water, the block needs to displace twice the volume of fluid in the second fluid to achieve the same buoyant force. The 30-N block, on the other hand, displaces the same amount of fluid in each case. However, it experiences twice the buoyant force from the water as it does from the second fluid because of the factor of two difference in the densities.

What happens with the 20-N block is particularly interesting, because it floats in water and yet sinks in the second fluid. This raises the question, what determines whether an object floats or sinks when it is placed in a fluid?

EXPLORATION 9.4 – Float or sink?

How can we tell whether an object will float or sink in a particular fluid? As we have considered before, when an object floats in a fluid the upward buoyant force exactly balances the downward force of gravity. This gives: $F_B = mg$.

Using Archimedes' principle, we can write the left-hand side as: $\rho_{fluid} V_{disp} g = mg$.

The factors of g cancel (this tells us that it doesn't matter which planet we're on, or where on the planet we are), giving: $\rho_{fluid} V_{disp} = m$.

If we write the right-hand side in terms of the density of the object, we get, for a floating object:

$$\rho_{fluid} V_{disp} = \rho_{object} V_{object} .$$

Re-arranging this equation leads to the interesting result (that applies for floating objects only):

$$\frac{\rho_{object}}{\rho_{fluid}} = \frac{V_{disp}}{V_{object}} . \qquad \text{(Equation 9.4: \textbf{For floating objects})}$$

Equation 9.4 answers the question of what determines whether an object floats or sinks in a fluid – the density. ***If an object is less dense than the fluid it is in then it floats***. An object that is less dense than the fluid it is in floats because the object displaces a volume of fluid smaller than its own volume – in other words, the object floats with part of it sticking out above the surface of the fluid. On the other hand, an object more dense than the fluid it is in must displace a volume of fluid larger than its own volume in order to float. This is certainly not possible for the solid blocks we have considered above. Thus, we can conclude that ***an object with a density larger than the density of the fluid it is in will sink in that fluid***.

Key Ideas: Whether an object floats or sinks in a fluid depends on its density. An object with a density less than that of a fluid floats in that fluid, while an object with a larger density than that of a fluid will tend to sink in that fluid. **Related End-of-Chapter Exercises: 1, 13.**

Equation 9.4 tells us that we can determine the density of a floating object by observing what fraction of its volume is submerged. For instance, if an object is 30% submerged in a fluid its density is 30% of the density of the fluid. Table 9.1 shows the density of various materials.

Material	Density (kg/m^3)	Material	Density (kg/m^3)
Interstellar space	10^{-20}	Planet Earth (average)	5500
Air (at 1 atmosphere)	1.2	Iron	7900
Water (at 4˚C)	1000	Mercury (the metal)	13600
Sun (average)	1400	Black hole	10^{+19}

Table 9.1 The density of various materials.

What about an object that has the same density as the fluid it is in? This is known as **neutral buoyancy**, because the upward buoyant force on the object balances the downward force of gravity on the object when the object is 100% submerged. Because the net force acting on the object is zero it is in equilibrium at any of the positions shown in Figure 9.14. This is true as long as the fluid density does not change with depth, which is something of an idealization. Again we are using a model, with an assumption of the model being that a fluid is incompressible – its density is constant.

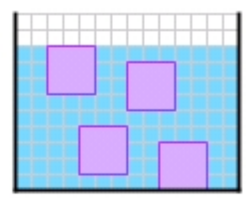

Figure 9.14: A neutrally buoyant object (an object with the same density as the surrounding fluid) will be at equilibrium at any of the positions shown. All other objects will either float at the surface, or sink to the bottom.

The general method for solving a typical buoyancy problem is based on the method we used in chapter 3 for solving a problem involving Newton's Laws. Now, we include Archimedes' principle. In general buoyancy problems are 1-dimensional, involving vertical forces, so that simplifies the method a little.

A General Method for Solving a Buoyancy Problem

1. Draw a diagram of the situation.
2. Draw one or more free-body diagrams, with each free-body diagram showing all the forces acting on an object as well as an appropriate coordinate system.
3. Apply Newton's Second Law to each free-body diagram.
4. If necessary, bring in Archimedes' principle, $F_B = \rho_{fluid} V_{disp} g$.
5. Put the resulting equations together and solve.

Essential Question 9.4: Let's say the four objects shown in Figure 9.14 have densities larger than that of the fluid. Can any of the objects be at equilibrium at the positions shown? Explain.