Answer to Essential Question 9.1: Object A. When an object floats in equilibrium, the buoyant force exactly balances the force of gravity. Object A displaces more fluid, so it experiences a larger buoyant force. This must be because object A weighs more than object B.

9-2 Using Force Methods with Fluids

EXAMPLE 9.2 – A block on a string

A block of weight $mg = 45$ N has part of its volume submerged in a beaker of water. The block is partially supported by a string of fixed length that is tied to a support above the beaker. When 80% of the block's volume is submerged, the tension in the string is 5.0 N.

(a) What is the magnitude of the buoyant force acting on the block?

(b) Water is steadily removed from the beaker, causing the block to become less submerged. The string breaks when its tension exceeds 35 N. What percent of the block's volume is submerged at the moment the string breaks?

(c) After the string breaks, the block comes to a new equilibrium position in the beaker. At equilibrium, what percent of the block's volume is submerged?

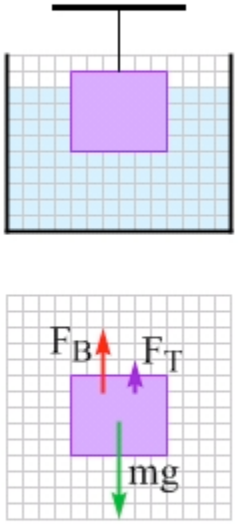

Figure 9.7: A diagram and a free-body diagram for the 45 N block floating in the beaker of water while partly supported by a string.

SOLUTION

As usual, we should begin with a diagram of the situation. A free-body diagram is also very helpful. These are shown in Figure 9.7.

(a) On the block's free-body diagram, we draw a downward force of gravity, applied by the Earth. We also draw an upward force of tension (applied by the string), and, because the block displaces some fluid, an upward buoyant force (applied by the fluid). The block is in equilibrium, so there must be no net force acting on the block.

Taking up to be positive, applying Newton's Second Law gives: $\sum \vec{F} = 0$.

Evaluating the left-hand side with the aid of the free-body diagram gives: $+F_T + F_B - mg = 0$.

Solving for the buoyant force gives: $F_B = mg - F_T = +45 \text{ N} - 5.0 \text{ N} = +40 \text{ N}$.

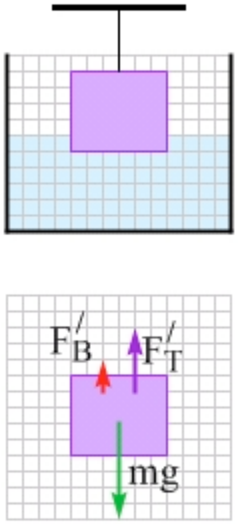

Figure 9.8: A diagram and free-body diagram of the situation just before the string breaks.

(b) As shown in Figure 9.8, removing water from the beaker causes the block to displace less fluid, so the magnitude of the buoyant force decreases. The magnitude of the tension increases to compensate for this. Applying Newton's Second Law again gives us essentially the same equation as in part (a). We can use this to find the new buoyant force, F_B'. Just before the string breaks we have:

$$F_B' = mg - F_T' = +45 \text{ N} - 35 \text{ N} = +10 \text{ N}.$$

Now, we can apply the idea that the buoyant force is proportional to the volume of fluid displaced. If a buoyant force of 40 N corresponds to a displaced volume equal to 80% of the block's volume, a buoyant force of 10 N (1/4 of the original force) must correspond to a displaced volume equal to 20% of the block's volume (1/4 of the original displaced volume).

(c) After the string breaks and the block comes to a new equilibrium position, we have a simpler free-body diagram, as shown in Figure 9.9. The buoyant force now, F_B'', applied to the block by the fluid, must balance the force of gravity applied to the block by the Earth. This comes from applying Newton's Second Law:

$$\sum \vec{F} = 0 .$$

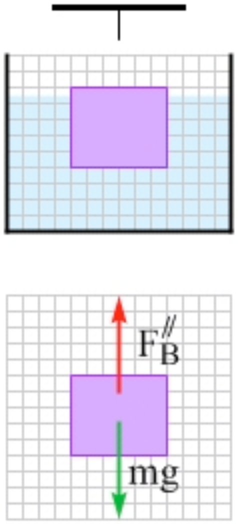

Figure 9.9: A diagram and free-body diagram for the situation after the string breaks, when the block has come to a new equilibrium position in the beaker.

Taking up to be positive, evaluating the left-hand side with the aid of the free-body diagram gives:

$$F_B'' - mg = 0 \text{, so } F_B'' = mg = 45 \text{ N} .$$

Using the same logic as in (b), if a buoyant force of 40 N corresponds to a displaced volume equal to 80% of the block's volume, a buoyant force of 45 N must correspond to a displaced volume equal to 90% of the block's volume.

Related End-of-Chapter Exercises: 21, 36.

Let's now extend our analysis to objects that sink. First, hang a block from a spring scale (a device that measures force) to measure the force of gravity acting on the block. With the block hanging from the spring scale, the scale reads 10 N, so there is a 10 N force of gravity acting on the block. A diagram and two free-body diagrams (one for the spring scale and one for the block) are shown in Figure 9.10.

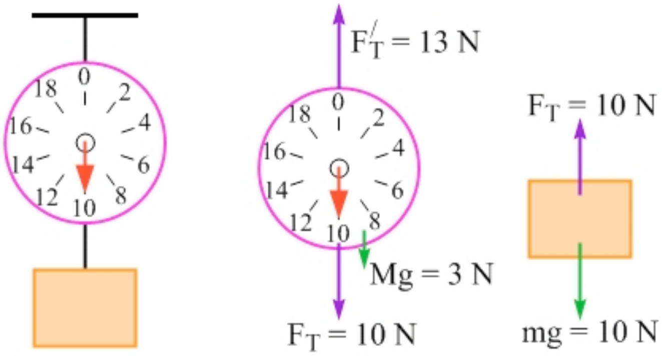

Figure 9.10: A diagram showing a block hanging from a spring scale, as well as free-body diagrams for the spring scale (which itself has a force of gravity of 3 N acting on it) and the block.

Question: With the block still suspended from the spring scale, let's dip the block into a beaker of water until it is exactly half submerged. Make a prediction. As we lower the block into the water, will the reading on the spring scale increase, decrease, or stay the same? Briefly justify your prediction.

Answer: The reading on the spring scale should decrease. This is because the spring scale no longer has to support the entire weight of the block. The more the block is submerged, the larger the buoyant force, and the smaller the spring-scale reading.

Essential Question 9.2: The spring scale reads 10 N when the block is out of the water. Let's say it reads 6.0 N when exactly 50% of the block's volume is below the water surface. What will the scale read when the entire block is below the water surface? Why?