Answer to Essential Question 9.10: The new radius is 95% of the original radius. Taking a factor of 0.95 to the fourth power gives approximately 0.81, so the new flow rate would only be 81% of the original flow rate. The flow rate would not drop as much if the pressure difference between the ends of the blood vessel increased. This would generally be accomplished by increasing the blood pressure (which can lead to health issues, of course).

9-11 Drag and the Ultracentrifuge

When an object is falling through a viscous fluid, a drag force acts on it. Unlike the kinetic friction force we looked at earlier in the book, which has a magnitude that is independent of speed, the viscous drag force is generally proportional to the speed, and opposite in direction to the velocity. This is known as Stokes' drag, with the drag force on a spherical particle of radius r moving at speed v through a fluid of viscosity η being:

$$F_d = -6\pi\eta r v. \qquad \text{(Equation 9.11: \textbf{Stokes' drag force for a spherical particle})}$$

When an object falls through the fluid, it will reach a terminal velocity when the drag force plus the buoyant force is equal and opposite to the force of gravity. In general, the smaller the object, the smaller the magnitude of the terminal velocity. Very small objects fall very slowly. If your goal is to separate particles from the fluid the particles are in, this can be a problem - it can take a long time for the particles to settle out at the bottom.

This is where an ultracentrifuge comes in. The job of the ultracentrifuge is to spin the fluid very quickly in a circular path. In that case, the effect is just like increasing the value of g by a large factor. Effectively, as far as the particles are concerned, they are in a gravitational field with a strength given by the centripetal acceleration, $\omega^2 r$. Spinning very quickly gives very large values of the angular speed (ω), leading to very large "effective gravity" that separates out the particles quickly and efficiently. Let's consider an example.

EXAMPLE 9.11 – Analyzing a blood sample

You get a blood sample drawn while you're seeing your doctor, and the sample is sent to the lab for analysis. A key part of the analysis involves running the sample (contained in a cylindrical tube) through an ultracentrifuge to separate out the components, which have different densities (the red blood cells being most dense, at 1125 kg/m^3, and the plasma being least dense, at 1025 kg/m^3). The average density of blood is about 1060 kg/m^3. What is the purpose of an ultracentrifuge, which, say, has a rotation rate of 5000 rpm and an acceleration 5000 times larger than the acceleration due to gravity? Why don't they just stand the tube of blood up vertically to let gravity separate it? Do a quantitative analysis, using the following values. The mass of a red blood cell is about 27×10^{-15} kg, the viscosity of blood is about 3.5×10^{-3} Pa s, and we will model the cell as a sphere of radius 3.5×10^{-6} m .

SOLUTION

We'll start by determining what gravity can do by itself. A red blood cell in a vertical tube of blood will reach a terminal velocity (v_t) when the drag force plus the buoyant force is equal and opposite to the force of gravity.

$$mg = \rho_{fluid} V g + 6\pi\eta r v_t \, .$$

We can replace the volume of the cell by $V = m / \rho_{cell}$, which gives

$$mg = \frac{\rho_{fluid}}{\rho_{cell}} mg + 6\pi\eta r v_t \, .$$

This re-arranges to $v_t = \left(1 - \frac{\rho_{fluid}}{\rho_{cell}}\right)\frac{mg}{6\pi\eta r}$, solving for the terminal speed of a blood cell.

Now, we'll substitute the relevant values into our equation (recognizing that our model has some limitations, such as that red blood cells are not spheres, and issues with the fluid density not being constant).

$$v_t = \left(1 - \frac{\rho_{fluid}}{\rho_{cell}}\right)\frac{mg}{6\pi\eta r} = \left(1 - \frac{1060 \text{ kg/m}^3}{1125 \text{ kg/m}^3}\right)\frac{\left(27\times 10^{-15} \text{ kg}\right)\left(9.8 \text{ N/kg}\right)}{6\pi\left(3.5\times 10^{-3} \text{ Pa s}\right)\left(3.5\times 10^{-6} \text{ m}\right)} = 6.6\times 10^{-8} \text{ m/s.}$$

This is very slow, of course. If we needed to wait until the red blood cell traveled a distance of 3.3 cm through the tube of blood, say, we would have to wait for 500000 s, which is approximately 6 days.

In our ultracentrifuge, where the acceleration is 5000 *g*, what is the difference? We use the same equation we derived above for the terminal speed, but we replace the factor of *g* by 5000 *g*. That increases the terminal velocity by a factor of 5000, and reduces the time it takes the blood cell to travel 3.3 cm by a factor of 5000, so the time would be 100 s instead of 500000 s (about 2 minutes instead of 6 days). Despite the simplifying assumptions in our model, this is in keeping with the recommendations of ultracentrifuge manufacturers, that a spin time of five minutes at 5000 *g* is appropriate. Our analysis gives a value of the same order of magnitude.

Where does the 5000 *g* come from? This goes back to uniform circular motion, in which the acceleration is given by:

$$a_c = \frac{v^2}{R} = \omega^2 R.$$

R here is the radius of the circular path traveled by a blood cell inside the centrifuge (not to be confused with the *r* used above, for the radius of the blood cell itself). Expressing the angular speed in rad/s, and the acceleration in meters per second, we could solve for *R*, for instance.

Related End-of-Chapter Exercises: 67 - 71.

Essential Question 9.11: A manufacturer has two different centrifuges, one that spins at 5000 rpm, and another that spins at 10000 rpm. Everything else is the same. The manufacturer makes the following claim about the faster centrifuge - "It costs three times as much, but it is also three times as efficient - you can run samples through the faster centrifuge in one third the time!" Based on our analysis above, the factor of three seems somewhat surprising - what would we expect the difference to be between the two models? Accounting for the fact that it takes some time for the centrifuge to reach its maximum angular speed, and to slow down to a stop at the end of a run, might this factor of three actually be plausible?