***Answer to Essential Question 9.9:*** These two situations appear to be different, but the answers are the same. In both cases the speed is given by an equation of the form $v = \sqrt{2gh}$. The equations, and the speeds, are the same because, in both cases, we can apply conservation of energy, with gravitational potential energy being transformed to kinetic energy.

## *9-10 Viscosity and Surface Tension*

Until this point, we have made a number of simplifying assumptions regarding the behavior of fluids, including assuming no viscosity (no resistance to flow). For any real fluid flowing through a pipe, there is some viscosity. In the case of air or water, the viscosity is generally low, but for applications such as blood flowing through blood vessels in the human body, the resistance to flow is an important factor.

Viscosity (a measure of a fluid's resistance to flow) is generally measured by means of a pair of horizontal parallel plates of area $A$ with fluid filling the space between them. The bottom plate is at rest, as is the fluid right next to it, while the top plate has a horizontal speed $v$, with the fluid next to it moving with the same velocity as that plate. This situation is known as Couette flow. Viscosity arises because neighboring layers of fluid have different speeds, and there is some frictional resistance between the layers as they move past one another. In general, the speed of the fluid increases linearly as you move from the fixed plate to the moving plate. In a Newtonian fluid, the viscosity ($\eta$) is a constant equal to the force required to keep the moving plate moving with constant velocity $v$ , multiplied by the length of the flow and divided by both the speed and the plate area. There are several classes of non-Newtonian fluids, in which the viscosity changes as the speed changes. Those are interesting, but beyond the scope of this book.

To give you some feel for typical numbers, the viscosity of water at 20˚C is 0.001002 Pa s, while that of motor oil is about 0.250 Pa s.

Unlike the somewhat artificial situation of Couette flow, in a typical application fluid is flowing through a pipe in which the pipe is stationary. In this case, the fluid next to the pipe wall is at rest, and the fluid at the center of the pipe is moving fastest. If we use $Q$ to denote the volume flow rate ($Q = Av$), then the volume flow rate of a viscous fluid is given by:

$$Q = \frac{\pi R^4 \Delta P}{8 \eta L} .$$ (Eq. 9.10: **The Hagen-Poiseuille equation**)

The equation is named for the German physicist and hydraulic engineer Gottlif Hagen and the French physician Jean Marie Louis Poiseuille, who independently arrived at the equation experimentally in 1839 and 1838, respectively.

The equation pertains to a fluid flowing through a pipe with a radius $R$ and a length $L$, with a pressure difference of $\Delta P$ between the ends of the pipe. The viscosity is denoted by $\eta$, the Greek letter eta.

**Surface Tension**

In general, we can't walk on water (at least not the liquid variety), and we also can't float a quarter on water. However, certain insects (water striders, for instance) can support themselves perfectly well on the surface of a pond, and it is possible to float a Japanese yen, or a metal paper clip, in a glass of water. Based on what we learned earlier, the yen coin or the paper clip should sink - they are each made from material that is denser than water - but they float because of surface tension. In essence, for these light objects, the water surface acts as an elastic membrane, much like the surface of a trampoline does for us.

The elastic nature of the water surface comes from the attraction the water molecules have for one another. A water strider will dent the surface, but (unlike us) will not break the surface. Similarly, the paper clip floating on the water surface dents the surface, much like we do when standing on a trampoline. It can break the surface and sink to the bottom - it can take a few tried to get it to float at the surface, but if you place it gently and carefully on the water surface, the force of gravity acting on it can be balanced by the force associated with surface tension. Adding some liquid soap to the water reduces its surface tension, so you can make the floating paper clip sink just by adding a few drops of soap. In addition to a paper clip, you can also float a Japanese yen coin, as shown in Figure 9.26.

**Figure 9.26:** A Japanese yen coin, made from aluminum, which is supported by the surface tension at the surface of the water in a glass of water. Note how the water surface is indented, with the surface acting much like an elastic membrane. Photo credit: A. Duffy.

You have probably noticed that water often tends to form drops. This is because there is energy associated with the surface, so surfaces generally take on the smallest area possible, to minimize that surface energy. A sphere is the shape that minimizes the surface area, for a given volume. A similar effect is seen with soap films, which can take on interesting shapes when a frame is drawn out of soapy water - the shape of the film tends to minimize the film's surface area.

**Surface tension and the lungs**

Surface tension is actually quite important for our breathing. First, think about blowing up a balloon. You have to work to fill the balloon, but if you don't hold the neck of the balloon then the balloon will simply deflate by itself, because of the balloon's surface tension. Our lungs have a very large number of tiny balloons, essentially - these are called alveoli. We use muscles to breathe in, inflating the alveoli. However, just like the balloon, the alveoli deflate all by themselves, to minimize surface tension. That's a key part of the breathing process.

The alveoli have a mucus coating on their walls, which acts as a ***surfactant*** (a material that reduces surface tension, like dish soap does when you're doing dishes). Unlike dish soap, however, which has a fixed surface tension, the mucus has a surface tension that increases with the size of the alveoli. This is important for a number of reasons. Having a low surface tension when the alveoli are small prevents surface tension from collapsing the alveoli, and surface tension increasing as the alveoli expand prevents the alveoli from getting too large. This also explains why premature babies often have breathing issues - their lungs do not have the mucus coating the alveoli to reduce the surface tension, making it difficult to inflate the alveoli.

***Essential Question 9.10:*** This question relates to viscosity. Let's say that Fred, who eats french fries on a daily basis, gradually experiences a narrowing of his blood vessels, because of plaque buildup in the vessel walls. All other things being equal, what would be the new flow rate through a blood vessel that experienced a 5% decrease in radius, compared to the original flow rate? In actuality, the blood pressure can change so that the flow rate does not drop quite so significantly. Would you expect the blood pressure to increase or decrease?