

**Answer to Essential Question 9.11:** If we double the angular velocity, the acceleration will increase by a factor of four (the acceleration goes as the square of the angular speed). That causes a corresponding decrease by a factor of 4 in the time to separate components of a sample - the manufacturer may be understating the case! However, the spin up and slow down time, which is probably longer for the faster centrifuge, means that the device is not consistently a factor of four better. That would decrease the ratio, although maybe not all the way down to a factor of three.

## Chapter Summary

### *Essential Idea for Fluids*

Even though situations involving fluids look quite different from those we examined in earlier chapters, we can apply the same methods we applied earlier. Forces are very useful for understanding situations in which an object floats or sinks in a static fluid, while energy-conservation ideas can help us to analyze situations involving moving fluids.

### *The Buoyant Force*

An object in a fluid experiences a net upward force we call the buoyant force,  $\vec{F}_B$ . The magnitude of the buoyant force is proportional to the volume of fluid displaced by the object.

$$F_B \propto V_{disp} \quad (\text{Equation 9.1: Buoyant force})$$

### *Mass Density*

Much of what happens with fluids involves mass density (often referred to simply as density). For instance, an object with a mass density larger than the mass density of the fluid it is in generally sinks in that fluid, while an object with a lower mass density than a fluid floats in the fluid. Using the symbol  $\rho$  for mass density, the relationship between mass, density, and volume is:

$$m = \rho V, \quad \text{or} \quad \rho = \frac{m}{V} \quad (\text{Eq. 9.2: Connection between mass and mass density})$$

### *Archimedes' Principle*

The magnitude of the buoyant force exerted on an object by a fluid is equal to the weight of the fluid displaced by the object. This is known as Archimedes' principle:

$$F_B = m_{disp} g = \rho_{fluid} V_{disp} g \quad (\text{Equation 9.3: Archimedes' Principle})$$

### *A General Method for Solving a Buoyancy Problem*

1. Draw a diagram of the situation.
2. Draw one or more free-body diagrams, with each free-body diagram showing all the forces acting on an object as well as an appropriate coordinate system.
3. Apply Newton's Second Law to each free-body diagram.
4. If necessary, bring in Archimedes' principle,  $F_B = \rho_{fluid} V_{disp} g$ .
5. Put the resulting equations together and solve.

### ***Pressure***

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} \quad \text{or} \quad P = \frac{F}{A}. \quad (\text{Equation 9.6: **Pressure**})$$

The MKS unit of pressure is the pascal (Pa).  $1 \text{ Pa} = 1 \text{ N/m}^2$ .

Standard atmospheric pressure is 101.3 kPa, or about  $1.0 \times 10^5$  Pa.

### ***Pressure in a static fluid***

$$P_2 = P_1 + \rho gh \quad . \quad (\text{Equation 9.7: **Pressure in a static fluid**})$$

### ***Fluid dynamics***

$$A_1 v_1 = A_2 v_2. \quad (\text{Equation 9.8: **The Continuity Equation**})$$

$$\rho gy_1 + \frac{1}{2} \rho v_1^2 + P_1 = \rho gy_2 + \frac{1}{2} \rho v_2^2 + P_2. \quad (\text{Equation 9.9: **Bernoulli's Equation**})$$

Bernoulli's Equation comes from applying energy-conservation ideas to fluids.

### ***Viscosity, and the drag force***

Viscosity is a measure of a fluid's resistance to flow, and arises because of friction between neighboring layers of fluid that are moving with different velocities.

If we use  $Q$  to denote the volume flow rate ( $Q = Av$ ), then the volume flow rate of a viscous fluid is given by:

$$Q = \frac{\pi R^4 \Delta P}{8\eta L}. \quad (\text{Eq. 9.10: **The Hagen-Poiseuille equation**})$$

The equation pertains to a fluid flowing through a pipe with a radius  $R$  and a length  $L$ , with a pressure difference of  $\Delta P$  between the ends of the pipe. The viscosity is denoted by  $\eta$ , the Greek letter eta.

The drag force on a spherical particle of radius  $r$  moving at speed  $v$  through a fluid of viscosity  $\eta$  is:

$$F_d = -6\pi\eta r v. \quad (\text{Equation 9.11: **Stokes' drag force for a spherical particle**})$$