Answer to Essential Question 8.5: Since $v_{escape} = \sqrt{\frac{2GM_E}{R_E}}$, keeping the mass the same while doubling the radius reduces the escape speed by a factor of $\sqrt{2}$.

8-6 Orbits

Imagine that we have an object of mass *m* in a circular orbit around an object of mass *M*. An example could be a satellite orbiting the Earth. What is the total energy associated with this object in its circular orbit?

The total energy is the sum of the potential energy plus the kinetic energy:

$$E = U + K = -\frac{GmM}{r} + \frac{1}{2}mv^2 .$$

This is a lovely equation, but it doesn't tell us much. Let's consider forces to see if we can shed more light on what's going on. For the object of mass *m* to experience uniform circular motion about the larger mass, it must experience a net force directed toward the center of the circle (i.e., toward the object of mass *M*). This is the gravitational force exerted by the object of mass *M*. Applying Newton's second law gives:

$\Sigma\vec{F} = m\vec{a} = \frac{mv^2}{r}$, directed toward the center.

$\frac{GmM}{r^2} = \frac{mv^2}{r}$, which tells us that $mv^2 = \frac{GmM}{r}$.

Substituting this result into the energy expression gives:

$$E = -\frac{GmM}{r} + \frac{GmM}{2r} = -\frac{GmM}{2r} .$$

This result is generally true for the case of a lighter object traveling in a circular orbit around a more massive object. We can make a few observations about this. First, the magnitude of the total energy equals the kinetic energy; the kinetic energy has half the magnitude of the gravitational potential energy; and the total energy is half of the gravitational potential energy. All this is true when the orbit is circular. Second, the total energy is negative, which is true for a **bound system** (a system in which the components remain together). Systems in which the total energy is positive tend to fly apart.

What happens when an object has a velocity other than that necessary to travel in a circular orbit? One way to think of this is to start the orbiting object off at the same place, with a velocity directed perpendicular to the line connecting the two objects, and simply vary the speed. If the speed necessary to maintain a circular orbit is denoted by $v_{\bigcirc}$, let's consider what happens if the speed is 20% less than $v_{\bigcirc}$; 20% larger than $v_{\bigcirc}$; the special case of $\sqrt{2}v_{\bigcirc}$; and $1.5v_{\bigcirc}$. The orbits followed by the object in these cases are shown in Figure 8.13.

Unless the object's initial speed is too small, causing it to eventually collide with the more massive object, an initial speed that is less than $v_{\bigcirc}$ will produce an elliptical orbit where the initial point turns out to be the farthest the object ever gets from the more massive object. The initial point is special because at that point the object's velocity is perpendicular to the gravitational force the object experiences.

If the initial speed is larger than $v_\bigcirc$, the result depends on how much larger it is. When the initial speed is $\sqrt{2}v_\bigcirc$ that is the escape speed, and is thus a special case. The shape of the orbit is parabolic, and this path marks the boundary between the elliptical paths in which the object remains in orbit and the higher-speed hyperbolic paths in which the object escapes from the gravitational pull of the massive object.

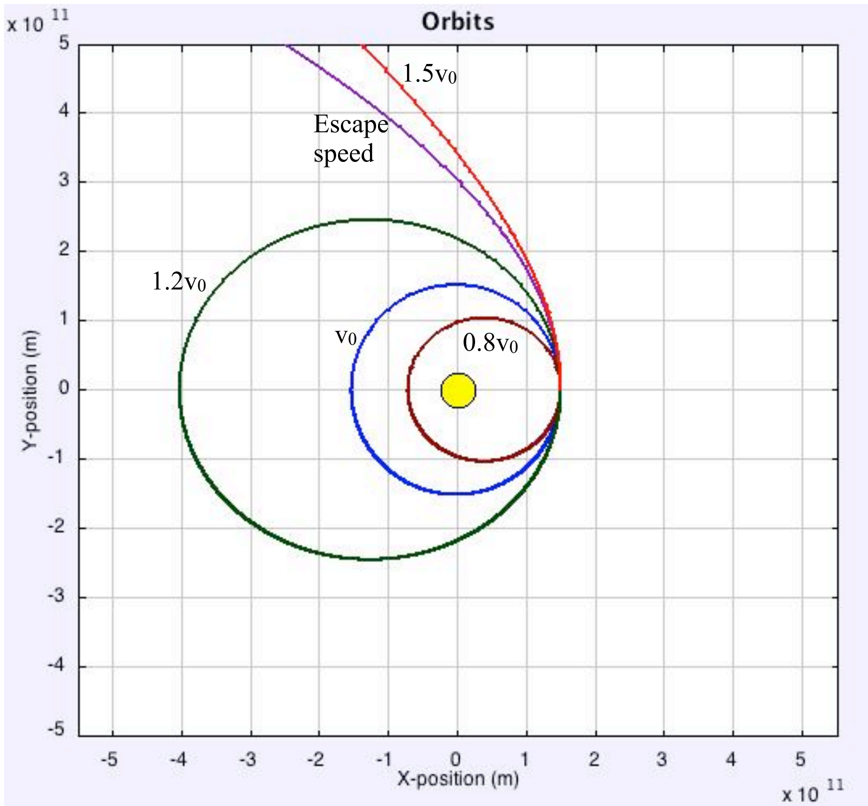

Figure 8.13: The orbits resulting from starting at a particular spot, the right-most point on each orbit, with initial velocities directed the same way (up in the figure) but with different initial speeds. The dark circular orbit represents the almost-circular orbit of the Earth, where the distances on each axis are in units of meters and the Sun is not shown but is located at the intersection of the axes. If the Earth's speed was suddenly reduced by 20%, the Earth would instead follow the smallest orbit, coming rather close to the Sun. If, instead, the Earth's speed was increased by 20%, the resulting elliptical orbit would take us quite a long way from the Sun before coming back again. Increasing the Earth's speed to $\sqrt{2}$ times its current speed (an increase of a little more than 40%) the Earth would be moving at the escape speed and we would follow the parabolic orbit to infinity (and beyond). Any initial speed larger than this would result in a hyperbolic orbit to infinity. Note that the speeds given in the picture represent initial speeds, the speed the Earth would have at the right-most point in the orbit to follow the corresponding path.

Related End-of-Chapter Exercises: 47, 59, and 60.

Essential Question 8.6: Is linear momentum conserved for any of these orbits? If so, which?