***Answer to Essential Question 8.4***: If we double each mass, the analysis above still works. Plugging $m$ = 2.0 kg into our speed equations shows that the speeds increase by a factor of $\sqrt{2}$ .

## *8-5 Example Problems*

**EXAMPLE 8.5A – Where is the field zero?**

Locations where the net gravitational field is zero are special, because an object placed where the field is zero experiences no net gravitational force. Let's place a ball of mass $m$ at the origin, and place a second ball of mass $9m$ on the $x$-axis at $x = +4a$. Find all the locations near the balls where the net gravitational field associated with these balls is zero.

**SOLUTION**

A diagram of the situation is shown in Figure 8.11. Let's now approach the problem conceptually. At every point near the balls there are two gravitational fields, one from each ball. The net field is zero only where the two fields are equal-and-opposite. These fields are in exactly opposite directions only at locations on the $x$-axis between the balls. If we get too close to the first ball it dominates, and if we get too close to the second ball it dominates; there is just one location between the balls where the fields exactly balance.

An equivalent approach is to use forces. Imagine having a third ball (we generally call this a **test mass**) and placing it near the other two balls. The third ball experiences two forces, one from each of the original balls, and these forces have to exactly balance. This happens at one location between the original two balls.

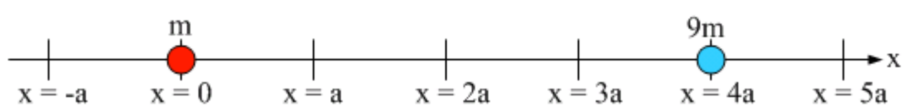


**Figure 8.11**: The two balls in Example 8.5A.

Whether we think about fields or forces, the approach is equivalent. The special place where the net field is zero is closer to the ball with the smaller mass. To make up for a factor of 9, representing the ratio of the two masses, we need to have a factor of 3 (which gets squared to 9) in the distances. In other words, we need to be three times further from the ball with a mass of $9m$ than we are from the ball of mass $m$ for the fields to be of equal magnitude. This occurs at x = $+a$.

We can also get this answer using a quantitative approach. Using the subscript 1 for the ball of mass $m$, and 2 for the ball of mass $9m$, we can express the net field as:

$$\vec{g}_{net} = \vec{g}_1 + \vec{g}_2 = 0.$$

Define right to be positive. If the point we're looking for is between the balls a distance $x$ from the ball of mass $m$, it is $(4a - x)$ from the ball of mass $9m$. Using the definition of $\vec{g}$ gives:

$$+\frac{Gm}{x^2} - \frac{G(9m)}{(4a-x)^2} = 0.$$

Canceling factors of $G$ and $m$, and re-arranging gives: $\frac{1}{x^2} = \frac{9}{(4a-x)^2}$.

Cross-multiplying leads to: $(4a-x)^2 = 9x^2$.

We could use the quadratic equation to solve for $x$, but let's instead take the square root of both sides of the equation. When we take a square root the result can be either plus or minus:

$$4a - x = \pm 3x.$$

Using the positive sign, we get $4a = +4x$, so $x = +a$. This is the correct solution, lying between the balls and closer to the ball with the smaller mass. Because it is three times farther from the ball of mass 9*m* than the ball of mass *m*, and because the distance is squared in the equation for field, this exactly balances the factor of 9 in the masses.

Using a minus sign gives a second solution, $x = -2a$. This location is three times farther (6*a*) from the ball of mass 9*m* than from the ball of mass *m* (2*a*). Thus at $x = -2a$ the two fields have the same magnitude, but they point in the same direction so they add rather than canceling.

**Related End-of-Chapter Exercises: 13, 14, 20.**

**EXAMPLE 8.5B – Escape from Earth**

When you throw a ball up into the air, it comes back down. How fast would you have to launch a ball so that it never came back down, but instead it escaped from the Earth? The minimum speed required to do this is known as the escape speed.

**SOLUTION**

A diagram is shown in Figure 8.12. Let's assume the ball starts at the surface of the Earth and that we can neglect air resistance (this would be fine if we were escaping from the Moon, but it is a poor assumption if we're escaping from Earth - let's not worry about that, however). We'll also assume the Earth is the only object in the Universe. So, this is an interesting calculation but the result will only be a rough approximation of reality.

Let's apply the energy conservation equation:

$$U_i + K_i + W_{nc} = U_f + K_f .$$

We're neglecting any work done by non-conservative forces, so $W_{nc} = 0$. The final gravitational potential energy is negligible, because the distance between the ball and Earth is very large (we can assume it to be infinite). What about the final kinetic energy? Because we're looking for the minimum initial speed let's use the minimum possible speed of the ball when it is very far from Earth, which we can assume to be zero. This leads to an equation in which everything on the right-hand side is zero:

$$U_i + K_i = 0 .$$

$$-\frac{GmM_E}{R_E} + \frac{1}{2}mv_{escape}^2 = 0 .$$

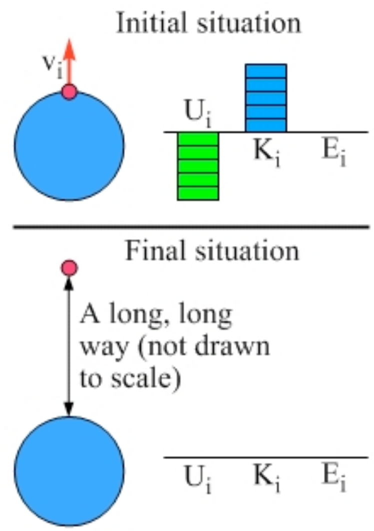


**Figure 8.12**: Energy bar graphs are shown in addition to the pictures showing the initial and final situations.

The mass of the ball does not matter, because it cancels out. This gives:

$$v_{escape} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2(6.67\times10^{-11}\ \text{N m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{6.37\times10^6\,\text{m}}} = 11.2\ \text{km/s} .$$

This is rather fast, and explains why objects we throw up in the air come down again!

**Related End-of-Chapter Exercises: 41, 42.**

***Essential Question 8.5:*** Let's say we were on a different planet that had the same mass as Earth but twice Earth's radius. How would the escape speed compare to that on Earth?