Answer to Essential Question 8.3: The direction of the gravitational field at a particular point is represented by the direction of the field vector at that point (or the ones near it if the point does not correspond exactly to the location of a field vector). The relative strength of the field is indicated by the darkness of the arrow. The larger the field's magnitude, the darker the arrow.

8-4 Gravitational Potential Energy

The expression we have been using for gravitational potential energy up to this point, $U_G = mgh$, applies when the gravitational field is uniform. In general, the equation for gravitational potential energy is:

$$U_G = -\frac{GmM}{r}.$$ (Equation 8.4: **Gravitational potential energy, in general**)

This gives the energy associated with the gravitational interaction between two objects, of mass m and M, separated by a distance r. The minus sign tells us the objects attract one another.

Consider the differences between the *mgh* equation for gravitational potential energy and the more general form. First, when using Equation 8.4 we are no longer free to define the potential energy to be zero at some convenient point. Instead, the gravitational potential energy is zero when the two objects are infinitely far apart. Second, when using Equation 8.4 we find that the gravitational potential energy is always negative, which is certainly not what we found with *mgh*. That should not worry us, however, because **what is critical is how potential energy changes** as objects move with respect to one another. If you drop your pen and it falls to the floor, for instance, both forms of the gravitational potential energy equation give consistent results for the change in the pen's gravitational potential energy.

Equation 8.4 also reinforces the idea that, when two objects are interacting via gravity, neither object has its own gravitational potential energy. Instead, gravitational potential energy is associated with the interaction between the objects.

EXPLORATION 8.4 – Calculate the total potential energy in a system

Three balls, of mass m, $2m$, and $3m$, are placed in a line, as shown in Figure 8.9. What is the total gravitational potential energy of this system?

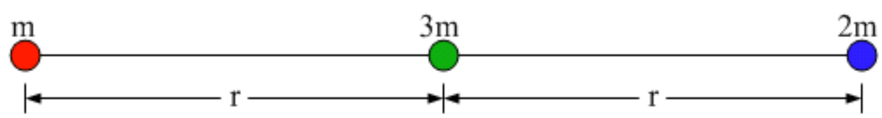

Figure 8.9: Three equally spaced balls placed in a line.

To determine the total potential energy of the system, consider the number of interacting pairs. In this case there are three ways to pair up the objects, so there are three terms to add together to find the total potential energy. Because energy is a scalar, we do not have to worry about direction. Using a subscript of 1 for the ball of mass m, 2 for the ball of mass $2m$, and 3 for the ball of mass $3m$, we get:

$$U_{Total} = U_{13} + U_{23} + U_{12} = -\frac{Gm(3m)}{r} - \frac{G(2m)(3m)}{r} - \frac{Gm(2m)}{2r} = -\frac{10Gm^2}{r}.$$

Key ideas for gravitational potential energy: Potential energy is a scalar. The total gravitational potential energy of a system of objects can be found by adding up the energy associated with each interacting pair of objects. **Related End-of-Chapter Exercises: 25, 29, 40.**

EXAMPLE 8.4 – Applying conservation ideas

A ball of mass 1.0 kg and a ball of mass 3.0 kg are initially separated by 4.0 m in a region of space in which they interact only with one another. When the balls are released from rest, they accelerate toward one another. When they are separated by 2.0 m, how fast is each ball going?

SOLUTION

Figure 8.10 shows the balls at the beginning and when they are separated by 2.0 m. Analyzing forces, we find that the force on each ball increases as the distance between the balls decreases. This makes it difficult to apply a force analysis. Energy conservation is a simpler approach. Our energy equation is:

$$U_i + K_i + W_{nc} = U_f + K_f \,.$$

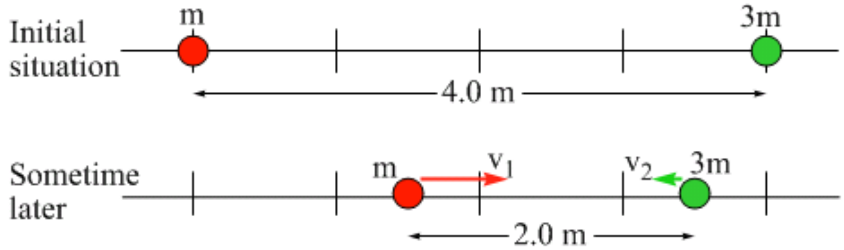

Figure 8.10: The initial situation shows the balls at rest. The force of gravity causes them to accelerate toward one another.

In this case, there are no non-conservative forces acting, and in the initial state the kinetic energy is zero because both objects are at rest. This gives $U_i = U_f + K_f$. The final kinetic energy represents the kinetic energy of the system, the sum of the kinetic energies of the two objects.

Let's solve this generally, using a mass of *m* and a final speed of v_1 for the 1.0 kg ball, and a mass of 3*m* and a final speed of v_2 for the 3.0 kg ball. The energy equation becomes:

$$-\frac{Gm(3m)}{4.0\text{ m}} = -\frac{Gm(3m)}{2.0\text{ m}} + \frac{1}{2}mv_1^2 + \frac{1}{2}(3m)v_2^2 \,.$$

Canceling factors of m gives: $-\frac{3Gm}{4.0\text{ m}} = -\frac{3Gm}{2.0\text{ m}} + \frac{1}{2}v_1^2 + \frac{3}{2}v_2^2$.

Multiplying through by 2, and combining terms, gives: $+\frac{3Gm}{2.0\text{ m}} = v_1^2 + 3v_2^2$.

Because there is no net external force, the system's momentum is conserved. There is no initial momentum. For the net momentum to remain zero, the two momenta must always be equal-and-opposite. Defining right to be positive, momentum conservation gives:

$0 = +mv_1 - 3mv_2$, which we can simplify to $v_1 = 3v_2$.

Substituting this into the expression we obtained from applying energy conservation:

$$+\frac{3Gm}{2.0\text{ m}} = (3v_2)^2 + 3v_2^2 = 12v_2^2$$

This gives $v_2 = \sqrt{\frac{Gm}{8.0\text{ m}}}$, and $v_1 = 3v_2 = 3\sqrt{\frac{Gm}{8.0\text{ m}}}$.

Using *m* = 1.0 kg, we get $v_2 = 2.9\times10^{-6}$ m/s and $v_1 = 8.7\times10^{-6}$ m/s.

Related End-of-Chapter Exercises: Problems 43 – 45.

Essential Question 8.4: Return to Example 8.4. If you repeat the experiment with balls of mass 2.0 kg and 6.0 kg instead, would the final speeds change? If so, how?