***Answer to Essential Question 8.1***: Newton's third law tells us that the gravitational force the Sun exerts on the Earth is equal in magnitude (and opposite in direction) to the gravitational force the Earth exerts on the Sun. This follows from Equation 8.1, because, whether we look at the force exerted by the Sun or the Earth, the factors going into the equation are the same.

8-2 The Principle of Superposition

EXPLORATION 8.2 – Three objects in a line

Three balls, of mass *m*, 2*m*, and 3*m*, are equally spaced along a line. The spacing between the balls is *r*. We can arrange the balls in three different ways, as shown in Figure 8.4. In each case the balls are in an isolated region of space very far from anything else.
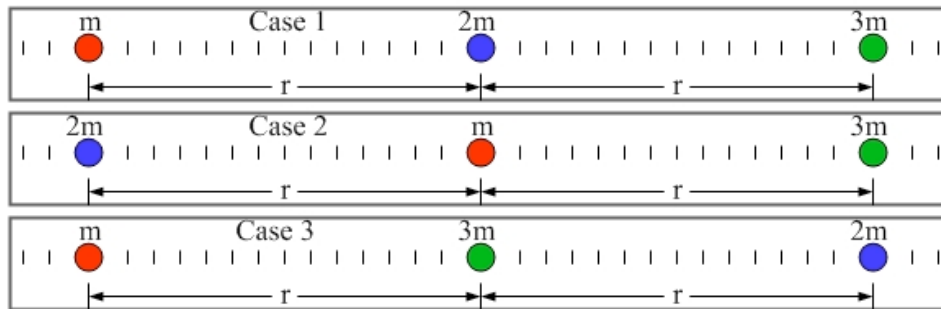

Figure 8.4: Three different arrangements of three balls of mass *m*, 2*m*, and 3*m* placed on a line with a distance *r* between neighboring balls.

Step 1 – ***How many forces does each ball experience in each case?*** Each ball experiences two gravitational forces, one from each of the other balls. We can neglect any other interactions.

Step 2 – ***Consider Case 1. Is the force that the ball of mass m exerts on the ball of mass 3m affected by the fact that the ball of mass 2m lies between the other two balls?***
Interestingly, no. To find the net force on any object, we simply add the individual forces acting on an object as vectors. This is known as **the principle of superposition**, and it applies to many different physical situations. In case 1, for instance, we find the force the ball of mass *m* applies to the ball of mass 3*m* as if the ball of mass 2*m* is not present. The net force on the ball of mass 3*m* is the vector addition of that force and the force on the 3*m* ball from the ball of mass 2*m*.

Step 3 – ***In which case does the ball of mass 2m experience the largest-magnitude net force? Argue qualitatively.*** Let's attach arrows to the ball of mass 2*m*, as in Figure 8.5, to represent the two forces the ball experiences in each case. The length of each arrow is proportional to the force.
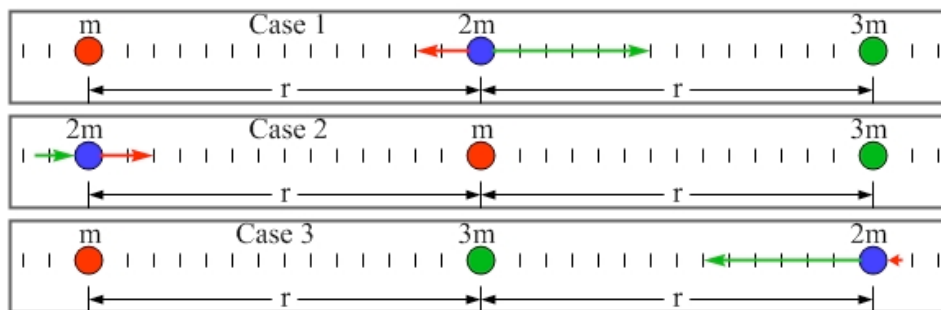
Figure 8.5: Attaching force vectors to the ball of mass 2*m*. The vectors point toward the object exerting the force. The length of each vector is drawn in units of Gm^2/r^2.

In case 1, the two forces partly cancel, and, in case 2, the forces add but give a smaller net force than that in case 3. Thus, the ball of mass $2m$ experiences the largest-magnitude net force in Case 3.

Step 4 – ***Calculate the force experienced by the ball of mass 2m in each case.***
To do this, we will make extensive use of Newton's Universal Law of Gravitation. Let's define right to be the positive direction, and use the notation $\vec{F}_{21}$ for the force that the ball of mass $2m$ experiences from the ball of mass m. In each case, $\vec{F}_{2,net} = \vec{F}_{21} + \vec{F}_{23}$.

Case 1: $\vec{F}_{2,net} = \vec{F}_{21} + \vec{F}_{23} = -\dfrac{Gm(2m)}{r^2} + \dfrac{G(2m)(3m)}{r^2} = -\dfrac{2Gm^2}{r^2} + \dfrac{6Gm^2}{r^2} = +\dfrac{4Gm^2}{r^2}$

Case 2: $\vec{F}_{2,net} = \vec{F}_{21} + \vec{F}_{23} = +\dfrac{Gm(2m)}{r^2} + \dfrac{G(2m)(3m)}{(2r)^2} = +\dfrac{2Gm^2}{r^2} + \dfrac{3Gm^2}{2r^2} = +\dfrac{7Gm^2}{2r^2}$

Case 3: $\vec{F}_{2,net} = \vec{F}_{21} + \vec{F}_{23} = -\dfrac{Gm(2m)}{(2r)^2} - \dfrac{G(2m)(3m)}{r^2} = -\dfrac{Gm^2}{2r^2} - \dfrac{6Gm^2}{r^2} = -\dfrac{13Gm^2}{2r^2}$

This approach confirms that the ball of mass $2m$ experiences the largest-magnitude net force in case 3.

Step 5 - ***Rank the three cases, from largest to smallest, based on the magnitude of the net force exerted on the ball in the middle of the set of three balls.*** Let's extend our pictorial method by attaching force vectors to each ball in each case, as in Figure 8.6.
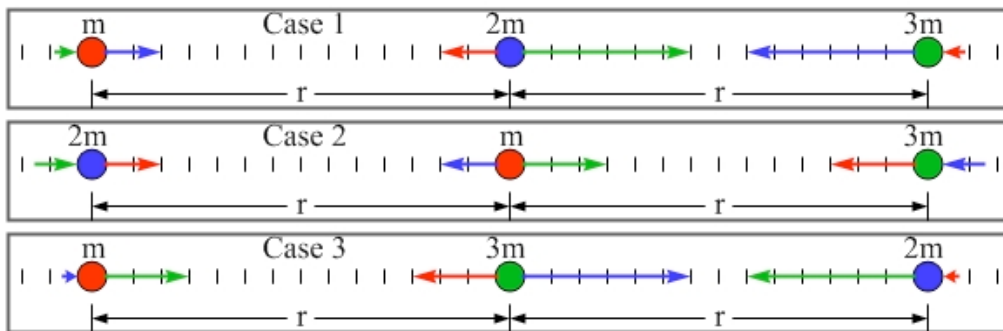
Figure 8.6: Attaching force vectors to the balls in each case. The force vectors point toward the ball applying the force. The length of each vector is drawn in units of Gm^2/r^2.

Again, when considering the net force on the middle ball, we need to add the individual forces as vectors. Referring to Figure 8.6, ranking the cases based on the magnitude of the net force exerted on the middle ball gives **Case 1 > Case 3 > Case 2**.

Key idea about the principle of superposition: The net force acting on an object can be found using the principle of superposition, adding all the individual forces together as vectors and remembering that each individual force is unaffected by the presence of other forces.
Related End of Chapter Exercises: 15 and 27.

Essential Question 8.2: In the Exploration above, which ball experiences the largest-magnitude net force in (i) Case 1 (ii) Case 2 (iii) Case 3?