

8-1 Newton's Law of Universal Gravitation

One of the most famous stories of all time is the story of Isaac Newton sitting under an apple tree and being hit on the head by a falling apple. It was this event, so the story goes, that led Newton to realize that the same force that brought the apple down on his head was also responsible for keeping the Moon in its orbit around the Earth, and for keeping all the planets of the solar system, including our own planet Earth, in orbit around the Sun. This force is the force of gravity.

It is hard to over-state the impact of Newton's work on gravity. Prior to Newton, it was widely thought that there was one set of physical laws that explained how things worked on Earth (explaining why apples fall down, for instance), and a completely different set of physical laws that explained the motion of the stars in the heavens. Armed with the insight that events on Earth, as well as the behavior of stars, can be explained by a relatively simple equation (see the box below), humankind awoke to the understanding that our fates are not determined by the whims of gods, but depend, in fact, on the way we interact with the Earth, and in the way the Earth interacts with the Moon and the Sun. This simple, yet powerful idea, that we have some control over our own lives, helped trigger a real enlightenment in many areas of arts and sciences.

The force of gravity does not require the interacting objects to be in contact with one another. The force of gravity is an attractive force that is proportional to the product of the masses of the interacting objects, and inversely proportional to the square of the distance between them.

A gravitational interaction involves the attractive force that any object with mass exerts on any other object with mass. The general equation to determine the gravitational force an object of mass M exerts on an object of mass m when the distance between their centers-of-mass is r is:

$$\vec{F}_G = -\frac{GmM}{r^2}\hat{r} \quad (\text{Equation 8.1: Newton's Law of Universal Gravitation})$$

where $G = 6.67 \times 10^{-11} \text{ N m}^2 / \text{kg}^2$ is known as the universal gravitational constant. The magnitude of the force is equal to GmM / r^2 while the direction is given by $-\hat{r}$, which means that the force is attractive, directed back toward the object exerting the force.

At the surface of the Earth, should we use $\vec{F}_G = m\vec{g}$ or Newton's Law of Universal Gravitation instead? Why is g equal to 9.8 N/kg at the surface of the Earth, anyway? The two equations must be equivalent to one another, at least at the surface of the Earth, because they represent the same gravitational interaction. If we set the expressions equal to one another we get:

$$mg = \frac{GmM}{r^2} \quad \text{which gives} \quad g = \frac{GM}{r^2} .$$

At the surface of the Earth M is the mass of the Earth, $M_E = 5.98 \times 10^{24} \text{ kg}$, and r is the radius of the Earth, $R_E = 6.37 \times 10^6 \text{ m}$. So, the magnitude of g at the Earth's surface is:

$$g_E = \frac{GM_E}{R_E^2} = \frac{(6.67 \times 10^{-11} \text{ N m}^2 / \text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{(6.37 \times 10^6 \text{ m})^2} = 9.83 \text{ N/kg}.$$

For any object at the surface of the Earth, when we use Newton's Law of Universal Gravitation, the factors G , M_E , and R_E are all constants, so, until this point in the book, we have simply been replacing the constant value of GM_E / R_E^2 by $g = 9.8 \text{ N/kg}$.

EXAMPLE 8.1 – A two-dimensional situation

Three balls, of mass m , $2m$, and $3m$, are placed at the corners of a square measuring L on each side, as shown in Figure 8.1. Assume this set of three balls is not interacting with anything else in the universe. What is the magnitude and direction of the net gravitational force on the ball of mass m ?

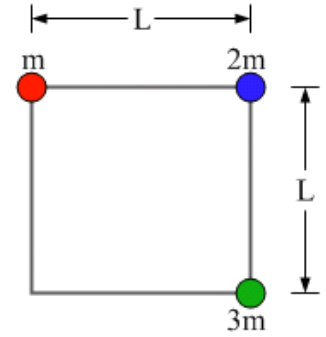


Figure 8.1: Three balls placed at the corners of a square.

SOLUTION

Let's begin by attaching force vectors to the ball of mass m . In Figure 8.2, each vector points toward the object exerting the force. The length of each vector is proportional to the magnitude of the force it represents.

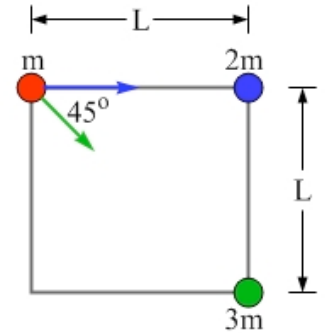


Figure 8.2: Attaching force vectors to the ball of mass m .

From the ball of mass $2m$: $\vec{F}_{21} = \frac{Gm(2m)}{L^2}$ to the right.

From the ball of mass $3m$: $\vec{F}_{31} = \frac{Gm(3m)}{L^2 + L^2}$ at 45° below the x-axis.

Finding the net force is a vector-addition problem.

In the x -direction, we get:

$$\vec{F}_{1x} = \vec{F}_{21x} + \vec{F}_{31x} = +\frac{2Gm^2}{L^2} + \frac{3Gm^2}{2L^2} \cos 45^\circ = \left(2 + \frac{3}{2\sqrt{2}}\right) \frac{Gm^2}{L^2}.$$

In the y -direction, we get: $\vec{F}_{1y} = \vec{F}_{21y} + \vec{F}_{31y} = 0 - \frac{3Gm^2}{2L^2} \sin 45^\circ = \left(-\frac{3}{2\sqrt{2}}\right) \frac{Gm^2}{L^2}.$

The Pythagorean theorem gives the magnitude of the net force on the ball of mass m :

$$F_1 = \sqrt{F_{1x}^2 + F_{1y}^2} = \sqrt{\left(4 + \frac{6}{\sqrt{2}} + \frac{9}{8} + \frac{9}{8}\right) \frac{Gm^2}{L^2}} = 3.24 \frac{Gm^2}{L^2}.$$

$$\text{The angle is given by: } \tan \theta = \frac{F_{1y}}{F_{1x}} = \frac{\frac{3}{2\sqrt{2}}}{\frac{4\sqrt{2} + 3}{2\sqrt{2}}} = \frac{3}{4\sqrt{2} + 3}.$$

So, the angle is 19.1° below the x -axis.

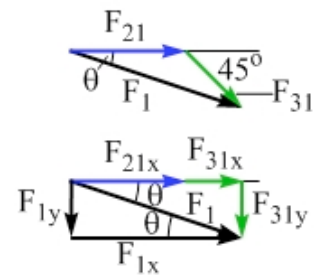


Figure 8.3: The triangle representing the vector addition problem being solved in Example 8.1.

Related End-of-Chapter Exercises: 16, 56 – 58.

Essential Question 8.1: The Sun has a much larger mass than the Earth. Which object exerts a larger gravitational force on the other, the Sun or the Earth?