End-of-Chapter Exercises

Exercises 1 – 12 are primarily conceptual questions that are designed to see if you have understood the main concepts of the chapter. Treat all balls with mass as point masses.

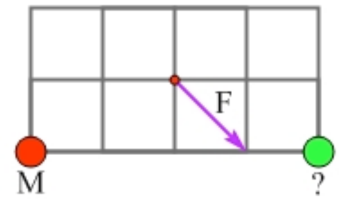


Figure 8.14: A small object experiences the net gravitational force shown. The ball on the right has an unknown mass. For Exercises 1 and 2.

1. Figure 8.14 shows the force a small object feels when it is placed at the location shown, near two balls. The ball on the left has a mass *M*, while the ball on the right has an unknown mass. Based on the force experienced by the small object, state whether the mass of the ball on the right is more than, less than, or equal to *M*. Justify your answer.

2. Return to the situation described in the previous problem, and shown in Figure 8.14. Determine the mass of the ball on the right, in terms of *M*.

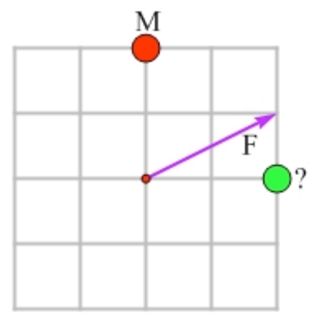
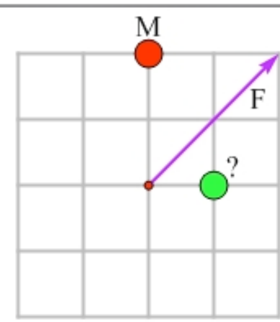


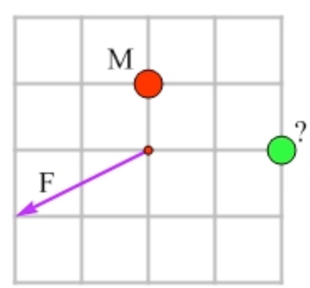
Figure 8.15: The two balls produce a net gravitational force directed up and to the right, as shown, on the object at the center of the diagram. For Exercise 3.

3. Figure 8.15 shows the net gravitational force experienced by a small object located at the center of the diagram. The force comes from two nearby balls, one with a charge of *M* and one with an unknown mass. (a) Is the mass of the second ball more than, less than, or equal to *M*? (b) Find the mass of the second ball.



4. Figure 8.16 shows the net gravitational force experienced by a small object located at the center of the diagram. The force comes from two nearby balls, one with a mass of *M* and one with an unknown mass. (a) Is the mass of the second ball more than, less than, or equal to *M*? (b) Find the mass of the second ball.

Figure 8.16: The two balls produce a net gravitational force at a 45° angle directed up and to the right, as shown, on the object at the center of the diagram. For Exercise 4.



5. Figure 8.17 shows the net gravitational force experienced by a small object located at the center of the diagram. The force comes from two nearby balls, one with a charge of *M* and one with an unknown mass. Find the mass of the second ball.

Figure 8.17: The two balls produce a net force directed down and to the left, as shown, on the object at the center of the diagram. For Exercise 5.

6. Two identical objects are located at different positions, as shown in Figure 8.18. The interaction between the objects themselves can be neglected. They experience forces of the same magnitude, and in the directions shown. Could these forces be produced by a single nearby object? If so, state where that object would be. If not, explain why not.

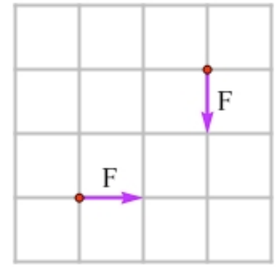


Figure 8.18: Two identical objects experience the forces shown in the diagram. The interaction between the objects themselves can be neglected. For Exercise 6.

7. Five balls, two of mass *m* and three of mass 2*m*, are arranged as shown in Figure 8.19. What is the magnitude and direction of the net gravitational force on the ball of mass *m* that is located at the origin?

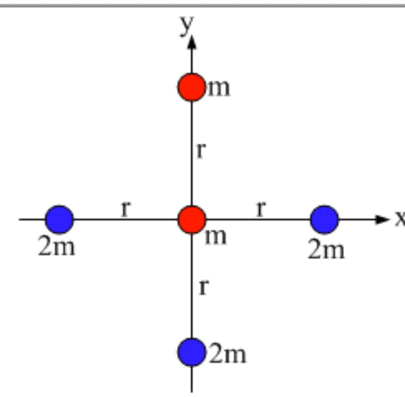


Figure 8.19: An arrangement of five balls, for Exercise 7.

8. Consider a region of space in which there is at least one object with mass. Can there be a location in this region where the net gravitational field is zero? Briefly justify your answer and, if you answer Yes, draw an arrangement of one or more objects that produces a zero gravitational field at a particular location.

9. You have four balls, two of mass *m* and two of mass 2*m*, and you will place one at each corner of a square. Show how you can arrange the balls so that the net gravitational field at the center of the square is (a) zero (b) directed to the right.

10. Let's say you were able to tunnel into the center of the Earth (and were somehow able to withstand the tremendous pressure and temperature). (a) What would be the magnitude of the gravitational force you would experience at the center of the Earth? (b) What does Equation 8.1 predict for the magnitude of this force? (c) Are your answers to (a) and (b) consistent with one another? Explain.

11. Which of the following is/are conserved for an object that is held in a circular orbit around another more massive object by the gravitational force between the objects? Justify your answers. (a) Kinetic energy? (b) Gravitational potential energy? (c) Total mechanical energy? (d) Linear momentum?

12. Repeat Exercise 11 in the situation where the orbit is elliptical rather than circular.

Exercises 13 – 17 deal with gravitational force.

13. Two balls are placed on the *x*-axis, as shown in Figure 8.20. The first ball has a mass *m* and is located at the origin, while the second ball has a mass 2*m* and is located at $x = +4a$. A third ball, with a mass of 4*m*, is then brought in and placed somewhere on the *x*-axis. Assume that each ball is influenced only by the other two balls. (a) Could the third ball be placed so that all three balls simultaneously experience no net force due to the other two? (b) Could the third ball be placed so that at least one of the three balls experiences no net force due to the other two? Briefly justify your answers.

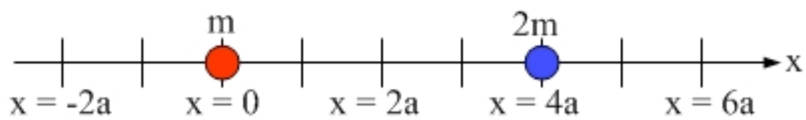


Figure 8.20: Two balls on the x-axis. These could represent planets you are placing in a solar system, in your role as designer of the cosmos. For Exercises 13 and 14.

14. Return to the situation described in Exercise 13, and find all the possible locations where the third ball could be placed so that at least one of the three balls experiences no net force due to the other two.

15. Refer back to Figure 8.9, showing a ball of mass 3*m* located halfway between a ball of mass *m* and a ball of mass 2*m*. Rank the three balls based on the magnitude of the net force they experience, from largest to smallest.

16. Three identical balls are arranged so there is one ball at each corner of an equilateral triangle. Each side of the triangle is exactly 1 meter long. If each ball experiences a net force of 5.00×10^{-6} N because of the other two balls, what is the mass of each ball?

17. Rank the four inner planets of the solar system (Mercury, Venus, Earth, and Mars) based on the magnitude of the gravitational force they each experience from the Sun, from largest to smallest.

Problems 18 – 23 deal with gravitational field.

18. Using the fact that the gravitational field at the surface of the Earth is about six times larger than that at the surface of the Moon, and the fact that the Earth's radius is about four times the Moon's radius, determine how the mass of the Earth compares to the mass of the Moon.

19. Five identical balls of mass M are placed so there is one ball at each corner of a regular pentagon. (a) If each ball is a distance R from the geometrical center of the pentagon, what is the magnitude of the gravitational field at the center of the pentagon due to the balls? (b) If the ball at the top of the pentagon is completely removed from the system, as shown in Figure 8.21, what is the magnitude and direction of the gravitational field at the center of the pentagon?

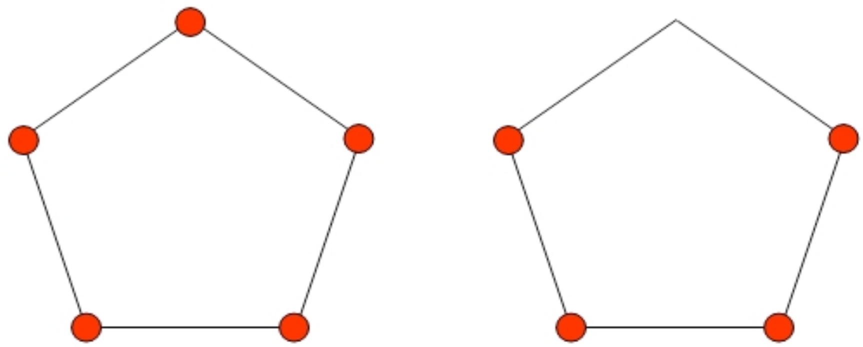


Figure 8.21: Initially, there are five identical balls, each placed at one vertex of a regular pentagon. The ball at the top is then removed, as shown at right. For Exercise 19.

20. At some point on the line connecting the center of the Earth to the center of the Moon the net gravitational field is zero. How far is this point from the center of the Earth?

21. A ball of mass $6m$ is placed on the x-axis at $x = -2a$. There is a second ball of unknown mass at $x = +a$. If the net gravitational field at the origin due to the two balls has a magnitude of $\frac{Gm}{a^2}$, what is the mass of the second ball? Find all possible solutions.

22. Repeat the previous exercise if the net gravitational field at the origin has a magnitude of $\frac{3Gm}{a^2}$.

23. A ball of mass $2m$ is placed on the x-axis at $x = -a$. There is a second ball with a mass of m that is placed on the x-axis at an unknown location. If the net gravitational field at the origin due to the two balls has a magnitude of $\frac{6Gm}{a^2}$, what is the location of the second ball? Find all possible solutions.

Exercises 24 – 31 deal with gravitational force, field, and potential energy.

24. A ball of mass $2m$ is placed on the x-axis at $x = -2a$. A second ball of mass m is placed nearby so that the net gravitational field at the origin because of the two balls is $\frac{Gm}{2a^2}$ in the negative x direction. Where is the second ball?

25. Consider the three cases shown in Exploration 8.2. (a) Rank these cases based on their gravitational potential energy, from most positive to most negative. (b) Determine the gravitational potential energy of the system shown in case 2.

26. Consider the three cases shown in Figure 8.22. Rank these cases, from largest to smallest, based on the (a) magnitude of the gravitational force experienced by the ball of mass *m*; (b) magnitude of the gravitational field at the origin; (c) gravitational potential energy of the system (do this ranking from most positive to most negative).

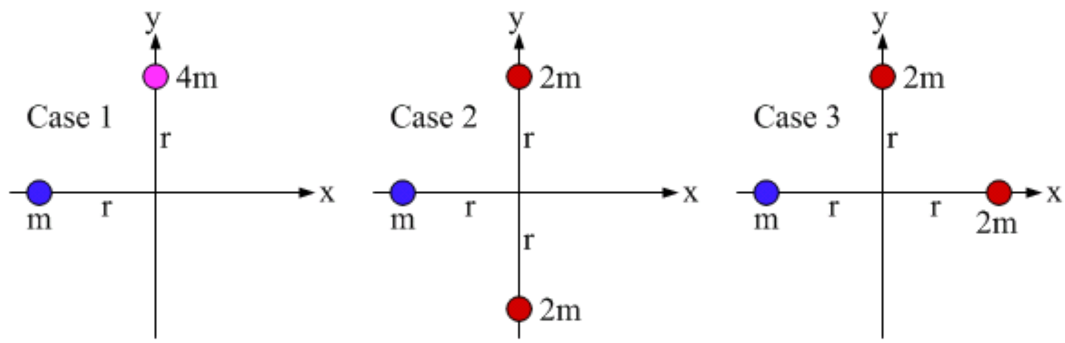


Figure 8.22: Three configurations of balls with mass, for Exercises 26 – 29.

27. Consider the three cases shown in Figure 8.22. Determine the magnitude and direction of the gravitational force experienced by the ball of mass *m* in (a) case1; (b) case 2; (c) case 3.

28. Consider the three cases shown in Figure 8.22. Determine the magnitude and direction of the gravitational field at the origin in (a) case1; (b) case 2; (c) case 3.

29. Consider the three cases shown in Figure 8.22. Determine the gravitational potential energy of the system in (a) case1; (b) case 2; (c) case 3.

30. A ball of mass 4*m* is placed on the *x*-axis at $x = +3a$ and a second ball of mass *m* is placed on the *x*-axis at $x = +6a$, as shown in Figure 8.23. (a) What is the gravitational potential energy associated with this system? (b) If you bring in a third ball of with a mass of 3*m* and place it at $x = +4a$, what is the gravitational potential energy of the three-ball system? (c) If the third ball had a mass of 2*m* instead what is the gravitational potential energy of the three-ball system?

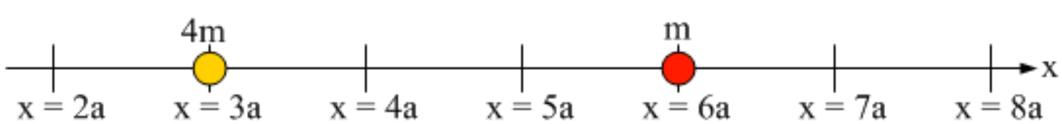


Figure 8.23: Two balls on the *x*-axis, a ball of mass 4*m* at *x* = +3a and a ball of mass *m* at *x* = +6a, for Exercises 30 – 31.

31. Consider again the system shown in Figure 8.253 (a) At how many locations near the two balls is the net gravitational field equal to zero? (b) Specify the locations of all such points.

General problems and conceptual questions

32. (a) Which object, the Sun or the Moon, exerts a larger gravitational force on the Earth? (b) By approximately what factor do these forces differ? (c) Is the Sun or the Moon primarily responsible for tides on the Earth? How do you explain this, given the answers to parts (a) and (b)?

33. Why do we need to have a leap year almost every four years? Sometimes we skip a leap year. Why would this be? What is the rule that determines which years are leap years and which years are skipped?

34. (a) If time zones were all one hour apart (this is not always the case), how many time zones would there be? Using the same assumption, what would the average width of a time zone be at (b) the equator? (c) a latitude equal to the latitude of Paris, France, which is 48.8° north?

35. (a) What is the speed of the Earth in its orbit around the Sun? (b) What is the acceleration of the Earth because of the gravitational force exerted on it by the Sun? (c) What is the acceleration of the Sun because of the gravitational force exerted on it by the Earth?

36. Two identical balls are placed some distance apart from one another. (a) Sketch a field vector diagram for this situation, assuming only the two balls contribute to the field. (b) Sketch a field line diagram for this situation.

37. Three balls, of mass m, $2m$, and $3m$, are arranged so there is one ball at each corner of an equilateral triangle. Each side of the triangle is exactly 1 meter long. (a) Rank the balls based on the magnitude of the net force they each experience, from largest to smallest. (b) Find the magnitude of the net force acting on the ball of mass $2m$.

38. Return to the situation described in Exercise 37. What is the magnitude of the gravitational field at the center of the triangle?

39. (a) Referring to Figure 8.24, which of the two balls of mass m experiences the larger net gravitational force? Justify your answer. (b) What is the magnitude of the net gravitational force experienced by the three different balls of mass $2m$?

40. Referring to Figure 8.24, what is the gravitational potential energy of this arrangement of five balls?

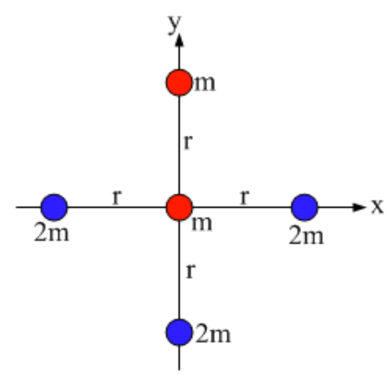


Figure 8.24: An arrangement of five balls, for Exercises 39 – 40.

41. (a) What is the escape speed for projectiles launched from the surface of the Moon? (b) Some people think that if we launch a manned mission to Mars it makes more sense to launch the spacecraft from the Moon rather than from the Earth. Comment on whether this makes sense, from an energy perspective.

42. If a projectile is launched straight up from the surface of the Moon with 90% of the speed necessary to escape from the Moon's gravity, what is the maximum distance it gets from the surface of the Moon before turning around? Assume the Moon is the only object influencing the projectile after launch.

43. Two identical objects of mass *m* are completely isolated from anything else and interact only with one another via gravity. If the two objects are both at rest when they are separated by a distance *L*, how fast are they each traveling when the distance between them is *L*/4?

44. Repeat the previous problem in the case when one object has a mass *m* and the second has a mass of 2*m*.

45. Two identical objects of mass 5.0 kg are initially 20 cm apart. They are then each given initial velocities of 0.10 m/s directed away from the other object. (a) Assuming they are completely isolated from anything else and interact only with one another via gravity, will they eventually come back together? (b) If so, determine their maximum separation distance; if not, determine how fast each object is traveling when they are very far apart.

46. Do some research on Johannes Kepler, and write two or three paragraphs explaining his contributions to our understanding of planetary orbits.

47. There is a Law known as Kepler's third law that states that when an object of mass *m* is held in a circular orbit around another object of mass *M* because of the gravitational interaction between them, the square of the period of the orbiting object is proportional to the cube of the orbital radius. Expressed as an equation, this is $T^2 = \frac{4\pi^2}{GM} r^3$. Let's derive the equation. (a) First, express the speed of the object of mass *m* in terms of the radius and period of the orbit. (b) Second, apply Newton's second law, using the fact that we're dealing with uniform circular motion. (c) Third, substitute your result from (a) into your result from (b) and re-arrange to get the result stated above.

48. Knowing that the Earth's orbit around the Sun is approximately circular with a radius of 150 million km, determine the mass of the Sun (see Exercise 47).

49. The speed of light is $3.0 \times 10^8\ m/s$. When Neil Armstrong and Buzz Aldrin landed on the Moon in July 1969, they left reflectors that would reflect a laser beam fired at the Moon from the Earth back to the Earth (see Figure 8.25). These reflectors are still used today. By measuring the round-trip time for the light scientists can determine the distance from the surface of the Earth to the surface of the Moon to within about 1 mm. (a) Assuming the laser is fired along the line connecting the centers of the Earth and Moon, and the round-trip time for the laser beam is measured to be 2.53 s, determine the center-to-center distance from the Earth to the Moon. The radius of the Earth is $6.38 \times 10^6\ m$, and the radius of the Moon is $1.74 \times 10^6\ m$. (b) Using the previous result, and knowing that the Moon takes 27.3 days to orbit the Earth, determine the mass of the Earth.

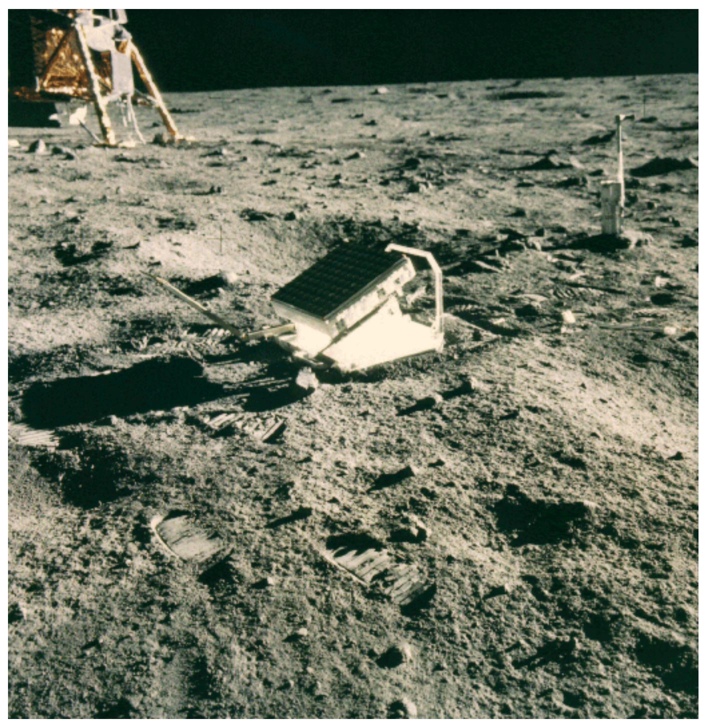


Figure 8.25: A photo of the set of 100 reflectors left by Neil Armstrong and Buzz Aldrin on the Moon in 1969, and described in Exercise 49. Photo courtesy of NASA'a Marshall Space Flight Center and [Science @ NASA.]

50. (a) Neglecting air resistance and the fact that the Earth is spinning, and assuming the ball does not hit anything in its travels, how fast would you have to launch a ball horizontally near the surface of the Earth so that it traveled in a circular path (with a radius equal to the radius of the Earth) around the Earth? (b) How long would the ball take to complete one orbit?

51. Some satellites are located in what is called a *geosynchronous* orbit around the Earth, in which they maintain their position over a particular location on the equator as the Earth spins on its axis (e.g., one might be over Ecuador at all times). How far from the center of the Earth is such a satellite (see Exercise 47), assuming it is over the equator?

52. The space shuttle orbits the Earth at an altitude that is typically 360 km above the Earth's surface. (a) What is the magnitude of the Earth's gravitational field at that altitude? (b) Explain why astronauts in the spaceship feel weightless.

53. A ball of mass $2m$ is placed on the x-axis at $x = -a$. There is a second ball with an unknown mass that is placed on the x-axis at an unknown location. If the force the second ball exerts on the first ball has a magnitude of $\frac{2Gm^2}{3a^2}$ and the gravitational potential energy associated with the interacting balls is $-\frac{2Gm^2}{a}$, what is the mass and location of the second ball? Find all possible solutions.

54. A ball of mass $3m$ is placed on the x-axis at $x = -a$. There is a second ball with an unknown mass that is placed on the x-axis at an unknown location. If the force the second ball exerts on the first ball has a magnitude of $\frac{Gm^2}{2a^2}$ and the net gravitational field at $x = 0$ due to these balls is $\frac{69Gm}{25a^2}$ in the positive x-direction, what is the mass and location of the second ball? Find all possible solutions.

55. A ball of mass $2m$ is placed on the x-axis at $x = -2a$. There is a second ball with an unknown mass that is placed on the x-axis at an unknown location. If the gravitational potential energy associated with the interacting balls is $-\frac{2Gm^2}{a}$ and the net gravitational field at $x = 0$ due to these balls has a magnitude of $\frac{Gm}{2a^2}$, what is the mass and location of the second ball? Find all possible solutions.

56. Four small balls are arranged at the corners of a square that measures L on each side, as shown in Figure 8.26. (a) Which ball experiences the largest-magnitude force due to the other three balls? (b) What is the direction of the net force acting on the ball with the mass of $4m$? (c) If you reduced the length of each side of the square by a factor of two, so neighboring balls were separated by a distance of $L/2$ instead, what would happen to the magnitude of the force experienced by each ball?

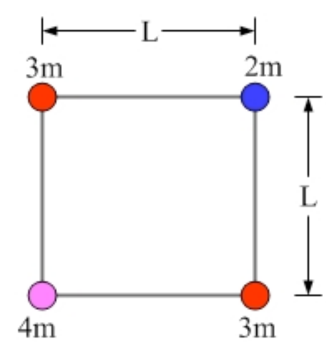


Figure 8.26: Four balls at the corners of a square, for Exercises 56 – 58.

57. Four small balls are arranged at the corners of a square that measures L on each side, as shown in Figure 8.26. Calculate the magnitude and direction of the force experienced by (a) the ball with the mass of $2m$, and (b) the ball with the mass of $3m$ in the lower right corner.

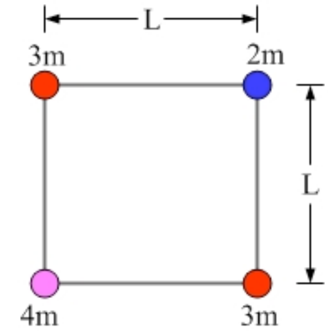


Figure 8.26: Four balls at the corners of a square, for Exercises 56 – 58.

58. Four small balls are arranged at the corners of a square that measures L on each side, as shown in Figure 8.26. (a) Calculate the magnitude and direction of the gravitational field at the center of the square that is produced by these balls. (b) Could you change the mass of just one of the balls to produce a net electric field at the center that is directed horizontally to the right? If so, which ball would you change the mass of and what would you change it to? If not, explain why not.

59. (a) What is the speed of the Earth in its approximately circular orbit around the Sun? (b) If the Earth's mass was suddenly reduced by a factor of 4, what would its speed have to be to maintain its orbit?

60. The four inner planets of the Solar System have orbits that are approximately circular. (a) Find the orbital speed of each of these four planets. (b) Rank the planets based on their orbital speed. Why is the ranking in this order?

61. What is the minimum amount of work required to move a satellite from a circular orbit around the Earth at an altitude of 350 km to a circular orbit at an altitude of 500 km?

62. (a) Look up the radius of the Sun, the distance from the Sun to the Earth, and the radius of the Earth. Use those numbers to determine the angle the Sun's diameter subtends if you were to look at the Sun (see Figure 8.29). (b) Repeat for the Moon, to determine the angle the Moon's diameter subtends when you look at the Moon. (c) Comment on the relative size of these angles.

63. Let's say you are an elf and are standing exactly at the North Pole looking due south at Santa's workshop. If you turn around 180˚ to face the other way, in which direction are you looking now?

64. Three students are having a conversation. Explain what you think is correct about what they say, and what you think is incorrect.

Anna: This question says that we have two objects, one with a mass of m and the other with a mass of 2m. It asks us for which one experiences a larger magnitude force because of the other object. That should be the smaller one, I think – shouldn't it feel twice as much force as the bigger one?

Mark: Well, the field created by the larger one should be twice as big as the field from the smaller one. Does that tell us anything?

Suzanne: But what about Newton's third law? Doesn't that say that any two objects, no matter how big, always exert equal-and opposite forces on one another?

Mark: I just kind of feel like the smaller one should feel more force.

Anna: I do, too, but what if we look at Newton's gravitation law? To get the force, you actually multiply the two masses together. So, it has to work out the same for each object.