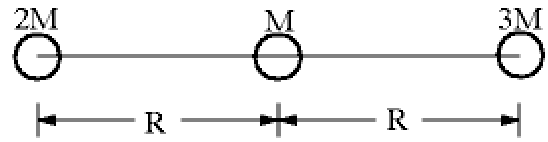
PROBLEM 1 – 10 points

[6 points] (a) Three masses, of mass 2*M*, *M*, and 3*M* are equally spaced along a line, as shown. The only forces each mass experiences are the forces of gravity from the other two masses.



[3 points] (i)Which mass experiences the largest magnitude net force?

[ ] mass 2*M* [ ] mass *M* [**X**] mass 3*M* [ ] equal for all three
[ ] the 2*M* and 3*M* masses have equal magnitude net forces larger than that of mass *M*

Justify your answer: **The forces on the middle object partly cancel because they point in opposite directions. In contrast, the 2*M* object feels two forces directed right, and the 3*M* object experiences two forces directed left. The 2*M* object exerts the same force on the 3*M* object that the 3*M* object exerts back on the 2*M* object, so the determining factor is the force exerted by the object of mass *M*. The object of mass *M* exerts a force on the object of mass 3*M* that is 50% larger than the force the object of mass *M* exerts on the 2*M* object, so the object of mass 3*M* experiences the largest net force.**

[3 points] (ii) What is the magnitude of the net force experienced by the object of mass 2*M*?

[ ] $GM^2/2R^2$ [ ] $2GM^2/R^2$ [ ] $3GM^2/R^2$ [**X**] $7GM^2/2R^2$ [ ] $8GM^2/R^2$

Justify your answer: **We can just add the forces from the other two objects. The net force on the 2M object is directed right with a magnitude of:**

$$F_{2M} = \frac{GM*2M}{R^2} + \frac{G*3M*2M}{(2R)^2} = \frac{4GMM}{2R^2} + \frac{3GMM}{2R^2} = \frac{7GM^2}{2R^2}$$

[4 points] (iii) Which of the following changes would cause the magnitude of the force experienced by the 2*M* object to increase by a factor of 4? **Select all that apply.**

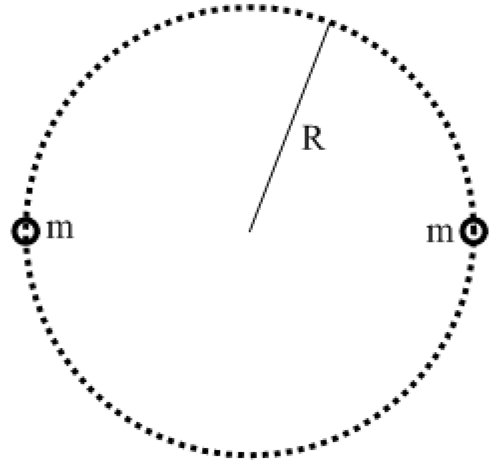
[**X**] double the mass of all three objects

[**X**] change the mass of the 2*M* object to 8*M*, without changing the mass of the other objects

[ ] double *R*

[**X**] Move the system to a parallel universe where the value of the universal gravitational constant is four times larger than its value in our universe

PROBLEM 2 – 10 points



A binary star system consists of two identical stars traveling in circular orbits of radius *R* around their center-of-mass. Each star travels at a constant speed *v*.

[7 points] (a) What is *m*, the mass of one of the stars? Express your answer in terms of *R*, *v*, and the universal gravitational constant *G*.

In this case, we can focus on just one of the stars, and apply Newton's second law to it. The only force on a star is the force of gravity from the other star – just be careful here to remember that the distance between the stars is 2*R*. We also recognize this as a case of uniform circular motion, so the acceleration is the centripetal acceleration

$$\sum F = ma = m\frac{v^2}{R}$$

$$\frac{Gmm}{(2R)^2} = m\frac{v^2}{R}$$

Cancel an m and an R from each side to get: $\frac{Gm}{4R} = v^2$ **so** $m = \frac{4Rv^2}{G}$

[3 points] (b) How much work does one star do on the other over half of an orbit?

The work done by one star is always zero. One way to see this is that the force on a star is always perpendicular to its displacement. Another way to see it is that the star's kinetic energy is constant.

PROBLEM 3 – 15 points

[8 points] (a) A ball of mass m is placed on the x-axis at $x = 0$. A second ball of mass $2m$ is placed on the x-axis at $x = -a$. A third ball, with a mass of m, is placed on the x-axis at an unknown location. If the net gravitational force exerted on the ball at the origin due to the other two balls has a magnitude of $\frac{6Gm^2}{a^2}$, what is the location of the third ball? Find all possible solutions.

Let's start with what we know. We know that the second ball exerts a force on the first ball that is $\frac{2Gm^2}{a^2}$ in the negative x direction. This gives us two possibilities for the third ball.

1. The net force on the first ball is in the negative x direction, so the third ball must apply a force of $\frac{4Gm^2}{a^2}$ in the negative x direction. If we say that the third ball is some distance x_1 to the left of the origin (to produce a force on the first ball in the negative direction), we get:

$$\frac{4Gm^2}{a^2} = \frac{Gm^2}{x_1^2} \quad \Rightarrow \quad \frac{4}{a^2} = \frac{1}{x_1^2}$$

This gives $x_1 = -\frac{a}{2}$ **.**

2. The net force on the first ball is in the positive x direction, so the third ball must apply a force of $\frac{8Gm^2}{a^2}$ in the positive x direction. If we say that the third ball is some distance x_2 to the right of the origin (to produce a force on the first ball in the positive direction), we get:

$$\frac{8Gm^2}{a^2} = \frac{Gm^2}{x_2^2} \quad \Rightarrow \quad \frac{8}{a^2} = \frac{1}{x_2^2}$$

This gives $x_2 = +\frac{a}{2\sqrt{2}}$ **.**

[3 points] (b) What is the numerical value of the escape speed for the Moon? In other words, with what speed does a projectile have to be launched from the surface of the Moon to escape from the Moon's gravity? The Moon's mass is $M = 7.35 \times 10^{22}$ kg and its radius is $R = 1.74 \times 10^6$ m.

Let's start with the energy-conservation equation: $K_i + U_i + W_{nc} = K_f + U_f$ **, where i represents the initial point, at the Moon's surface, and f represents the final point – in an escape speed situation, the final point is at infinity. We neglect any work done by non-conservative forces (this is a reasonable assumption, for the Moon), and in an escape speed situation the object just barely has to reach infinity (where the potential energy is zero), so the final kinetic energy is zero. The energy conservation equation can be reduced to:** $K_i + U_i = 0$ **.**

Expanding each of the terms, we get: $\frac{1}{2}mv_i^2 - \frac{GmM}{R} = 0$ **.**

Solving for the initial velocity gives: $v_i = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2 \times (6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2) \times (7.35 \times 10^{22} \text{ kg})}{1.74 \times 10^6 \text{ m}}}$ **.**

This gives 2370 m/s.

[4 points] (c) If a projectile is launched straight up from the surface of the Moon with 90% of the escape speed, what is the maximum distance it gets from the surface of the Moon before turning around? Assume the Moon is the only object influencing the projectile after launch.

Once again, we can start with the energy-conservation equation: $K_i + U_i + W_{nc} = K_f + U_f$ **, where i represents the initial point, at the Moon's surface, and f represents the final point – which in this case is the maximum distance the projectile gets from the surface of the Moon. Once again, we neglect any work done by non-conservative forces, and we set the final kinetic energy to zero.**

The energy conservation equation now reduces to: $K_i + U_i = U_f$ **.**

Expanding each of the terms, we get: $\frac{1}{2}mv_i^2 - \frac{GmM}{R} = -\frac{GmM}{R+h}$**, where h is what we're solving for.**

Our initial velocity in this case is $v_i = (0.9)\sqrt{\frac{2GM}{R}}$ **. Substituting this expression into the energy-conservation equation above gives:**

$$(0.81)\frac{GmM}{R} - \frac{GmM}{R} = -\frac{GmM}{R+h}$$

Simplifying this expression gives:

$$\frac{0.19}{R} = \frac{1}{R+h}$$

$$0.19(R+h) = R$$

$$h = \frac{0.81R}{0.19} = 7.42 \times 10^6 \text{ m}$$