Answer to Essential Question 7.6: A whole-vector approach, not splitting the velocity and momentum vectors into components, would also work (see End-of-Chapter Exercise 58).

7-7 Combining Energy and Momentum

To analyze some situations, we apply both energy conservation and momentum conservation in the same problem. The trick is to know when to apply energy conservation (and when not to!) and when to apply momentum conservation. Consider the following Exploration.

EXPLORATION 7.7 – Bringing the concepts together

Two balls hang from strings of the same length. Ball A, with a mass of 4.0 kg, is swung back to a point 0.80 m above its equilibrium position. Ball A is released from rest and swings down and hits ball B. After the collision, ball A rebounds to a height of 0.20 m above its equilibrium position and ball B swings up to a height of 0.050 m. Let's use $g = 10$ m/s^2 to simplify the calculations.

Step 1 – ***Sketch a diagram of the situation.*** This is shown in Figure 7.14.

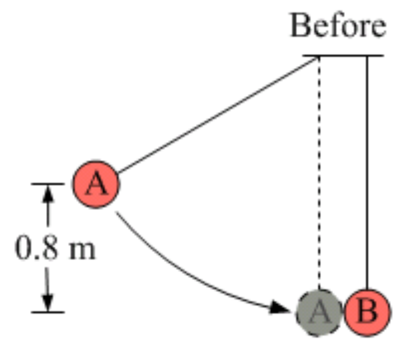

Figure 7.14: A diagram of the two balls on strings. Ball A is swung back until it is 0.80 m higher than its equilibrium point and released from rest.

Step 2 – ***Our goal is to find the mass of ball B. Can we find the mass by setting the initial gravitational potential energy of ball A equal to the sum of the final potential energy of ball A and the final potential energy of ball B? Explain why or why not.*** The answer to the question is no. We can use energy conservation to help solve the problem, but setting the mechanical energy before the collision equal to the mechanical energy after the collision is assuming too much. The balls make contact in the collision, so it is likely that some of the mechanical energy is transformed to thermal energy, for instance.

Step 3 – ***Apply energy conservation to find the speed of ball A just before the collision.*** The gravitational potential energy of ball A is transformed into kinetic energy just before the collision. We will neglect the work done by air resistance, so we can apply energy conservation before the collision. Let's start with the conservation of energy equation: $K_i + U_i + W_{nc} = K_f + U_f$.

For ball A's swing before the collision, we know that the initial kinetic energy is zero. We are assuming that non-conservative forces do no work. We can also define the zero level for gravitational potential energy to be the lowest point in the swing, just before A hits B, so $U_f = 0$. The five-term equation reduces to:

$$U_i = K_f ;$$

$$mgh = \frac{1}{2} m v_f^2 ;$$

So, $v_f = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.8} = \sqrt{16} = 4.0 \text{ m/s}$.

Step 4 – ***Apply conservation of energy again to find the speed of ball A just after the collision.*** We could try applying conservation of momentum here, but there are too many unknowns. Instead, we can follow the conservation of energy method we used above. Note that we will not state that the kinetic energy immediately before the collision is equal to the kinetic energy after the collision, because that is not true. We can apply energy conservation, however, if we confine ourselves to the mechanical energy before the collision (as in step 3) or to the mechanical energy after the collision (this step). If we focus on the upswing, we have the kinetic energy of ball A, immediately after the collision, being transformed into gravitational potential energy. The conservation of energy equation reduces to:

$K_i = U_f$;

$\frac{1}{2} m v_i^2 = mgh$;

So, $v_i = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.2} = \sqrt{4.0} = 2.0$ m/s .

Let's be clear on what we have calculated in parts 3 and 4, because the notation can be confusing. We are analyzing the motion in three separate parts. The first part of ball A's motion is the downswing, which we analyzed in step 3. The third part is the upswing, which we analyzed in step 4. The second part is the collision, which we still have to analyze. The velocity of ball A immediately before the collision, at the end of the downswing, is $\vec{v}_{Ai} = 4.0 \text{ m/s}$ to the right, while A's velocity just after the collision, at the start of the upswing, is $\vec{v}_{Af} = 2.0 \text{ m/s}$ to the left. These are the values we will use in the conservation of momentum equation in step 5.

Step 5 – ***First, apply energy conservation to find the speed of ball B after the collision. Then, apply momentum conservation to find the mass of ball B.*** We still have to find the velocity of ball B, after the collision, before we use the conservation of momentum equation to find ball B's mass. We can find B's speed immediately after the collision by following the same process we used for ball A in step 3. We get:

$K_i = U_f$

$\frac{1}{2} m v_i^2 = mgh$

So, $v_{Bf} = \sqrt{2gh_B} = \sqrt{2 \times (10 \text{ m/s}^2) \times 0.050 \text{ m}} = \sqrt{1.0 \text{m}^2/\text{s}^2} = 1.0 \text{ m/s}$.

The velocity of ball B immediately after the collision is $\vec{v}_{Bf} = 1.0 \text{ m/s}$ to the right.

Now, we can write out a conservation of momentum equation to solve for the mass of ball B. It is critical to account for the fact that momentum is a vector. In this case, we account for the vector nature of momentum by using a minus sign for the velocity of ball A after the collision to reflect that it is moving to the left, when we chose right to be the positive direction. This gives: $m_A \vec{v}_{Ai} + m_B \vec{v}_{Bi} = m_A \vec{v}_{Af} + m_B \vec{v}_{Bf}$, where $\vec{v}_{Bi} = 0$. Solving for the mass of ball B gives:

$$m_B = \frac{m_A \vec{v}_{Ai} - m_A \vec{v}_{Af}}{\vec{v}_{Bf}} = \frac{(4.0 \text{kg}) \times (+4.0 \text{ m/s}) - (4.0 \text{ kg}) \times (-2.0 \text{ m/s})}{+1.0 \text{ m/s}} = 24 \text{ kg} .$$

Key idea: In some situations, we can apply conservation of energy and conservation of momentum ideas together. In general, we apply conservation of momentum to connect the situation before the collision to the situation after the collision. We use energy conservation to learn something about the situation before the collision and/or the situation afterwards.
Related End-of-Chapter Exercises: 30 – 32.

Essential Question 7.7: Is the collision in Exploration 7.7 super-elastic, elastic, inelastic, or completely inelastic? Justify your answer in two different ways.