Answer to Essential Question 7.5: To conserve energy of the planet-probe system, the planet's speed must decrease when the probe's speed increases. Because the planet's mass is so much larger than the probe's, this decrease in speed is negligible. Thus, the assumption is reasonable.

7-6 Collisions in Two Dimensions

Momentum conservation also applies in two and three dimensions. The standard approach to a two-dimensional (or even three-dimensional) problem is to break the momentum into components and conserve momentum in both the *x* and *y* directions separately. For colliding objects, the conservation of momentum equation in the *x*-direction, for instance, is:

$\vec{p}_{1ix} + \vec{p}_{2ix} = \vec{p}_{1fx} + \vec{p}_{2fx}$. (Eq. 7.5: **Conserving momentum in the x-direction**)

This can be written in an equivalent form:

$m_1\vec{v}_{1ix} + m_2\vec{v}_{2ix} = m_1\vec{v}_{1fx} + m_2\vec{v}_{2fx}$ (Eq. 7.6: **Momentum conservation, *x*-direction**)

Similar equations apply in the *y*-direction.

EXPLORATION 7.6 – A two-dimensional collision

An object of mass *m,* moving in the +*x*-direction with a velocity of 5.0 m/s, collides with an object of mass 2*m*. Before the collision, the second object has a velocity given by $\vec{v}_{2i} = -3.0 \text{ m/s } \hat{x} + 4.0 \text{ m/s } \hat{y}$, while, after the collision, its velocity is 3.0 m/s in the +*y*-direction. What is the velocity of the first object after the collision?

Step 1 – ***Draw a diagram of the situation.*** This is shown in Figure 7.12.

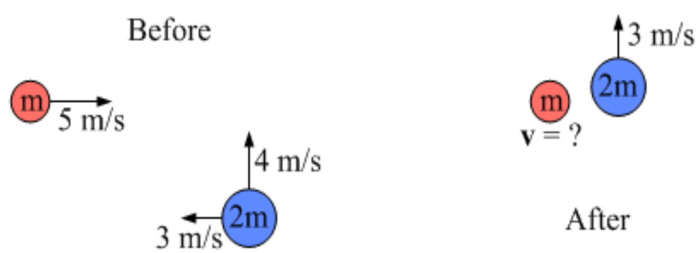

Figure 7.12: A diagram of the objects before and after they collide.

Step 2 - ***Set up a table showing the momentum components of each object before and after the collision.*** Organizing components into a table helps us keep the *x*-direction information separate from the *y*-direction information. We can combine the components into one vector at the end.

	***x*-direction**	***y*-direction**
Before the collision	$\vec{p}_{1ix} = m\vec{v}_{1ix} = +(5 \text{ m/s})m$	$\vec{p}_{1iy} = m\vec{v}_{1iy} = 0$
	$\vec{p}_{2ix} = 2m\vec{v}_{2ix} = -(6 \text{ m/s})m$	$\vec{p}_{2iy} = m\vec{v}_{2iy} = +(8 \text{ m/s})m$
After the collision	$\vec{p}_{1fx} = m\vec{v}_{1fx} = ?$	$\vec{p}_{1fy} = m\vec{v}_{1fy} = ?$
	$\vec{p}_{2fx} = 2m\vec{v}_{2fx} = 0$	$\vec{p}_{2fy} = 2m\vec{v}_{2fy} = +(6 \text{ m/s})m$

Table 7.2: Organizing the collision data in a table helps to keep the *x*-direction information separate from the *y*-direction information, and doing so can also help us solve the problem.

Step 3 – ***Apply conservation of momentum in the x-direction, and find the x-component of the first object's final velocity.*** Applying momentum conservation in the *x*-direction involves writing down the equation $\vec{p}_{1ix} + \vec{p}_{2ix} = \vec{p}_{1fx} + \vec{p}_{2fx}$.

This gives $\vec{p}_{1fx} = \vec{p}_{1ix} + \vec{p}_{2ix} - \vec{p}_{2fx} = +(5 \text{ m/s})m - (6 \text{ m/s})\, m - 0 = -(1 \text{ m/s})m$.

To get the velocity component in the x-direction we just divide by the mass, m.

$$\vec{v}_{1fx} = \frac{\vec{p}_{1fx}}{m} = \frac{-(1\text{ m/s})m}{m} = -1\text{ m/s}.$$

Step 4 – ***Use a similar process in the y-direction to find the y-component of the first object's final velocity.*** Applying momentum conservation in the y-direction involves writing down the equation $\vec{p}_{1iy} + \vec{p}_{2iy} = \vec{p}_{1fy} + \vec{p}_{2fy}$.

This equation gives $\vec{p}_{1fy} = \vec{p}_{1iy} + \vec{p}_{2iy} - \vec{p}_{2fy} = 0 + (8\text{ m/s})m - (6\text{ m/s})m = +(2\text{ m/s})m$.

To get the velocity component in the y-direction, we divide by the mass, m.

$$\vec{v}_{1fy} = \frac{\vec{p}_{1fy}}{m} = \frac{+(2\text{ m/s})m}{m} = +2\text{ m/s}.$$

Step 5 – ***Combine the x and y components to find the first object's final speed. Also, write down an expression for the first object's final velocity.*** We can use the Pythagorean theorem to find the final speed of the first object:

$$v_{1f} = \sqrt{v_{1fx}^2 + v_{1fy}^2} = \sqrt{1^2 + 2^2} = \sqrt{5}\text{ m/s}.$$

The velocity can be written in terms of components as $\vec{v}_{1f} = -1\text{ m/s}\,\hat{x} + 2\text{ m/s}\,\hat{y}$. The first ball's final velocity is shown in Figure 7.13.

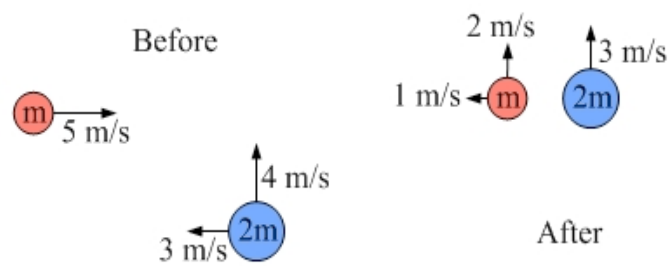

Figure 7.13: The situation Before and After the collision.

Key idea for momentum problems: We can solve a momentum problem in two dimensions with a strategy based on the independence of x and y, breaking a two-dimensional problem into two independent one-dimensional problems. **Related End-of-Chapter Exercises: 29, 57.**

Now that we've looked at a few examples, let's summarize a general method for solving a problem in which there is a collision.

A General Method for Solving a Problem That Involves a Collision

1. Draw a diagram of the situation, showing the velocity of the objects immediately before and immediately after the collision.
2. In a two-dimensional situation, set up a table showing the components of the momentum before and after the collision for each object.
3. Use momentum conservation: $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$. (Apply this twice, once for each direction, in a two-dimensional situation.) Account for the fact that momentum is a vector by using appropriate + and – signs.
4. If you need an additional relationship (such as in the case of an elastic collision), use the elasticity relationship or write an energy-conservation equation.

Essential Question 7.6: The strategy outlined above, which we applied in Exploration 7.6, relies on breaking vectors into components. Is there another method that we could use to solve the problem without using components?