Answer to Essential Question 7.4: For momentum to be conserved, either no net force is acting on the system, or the net force must act over such a small time interval that it has a negligible effect on the momentum of the system.

7-5 Classifying Collisions

If we attach Velcro to our colliding carts, and the carts stick together after the collision, as in Exploration 7.4, the collision is **completely inelastic**. If we remove the Velcro, so the carts do not stick together, we can set up a collision with the same initial conditions (cart 1 moving toward cart 2, which is stationary) and get a variety of outcomes. We generally classify these outcomes into four categories, depending on what happens to the kinetic energy in the collision.

We can also define a parameter k called the ***elasticity***. Elasticity is the ratio of the relative velocity of the two colliding objects after the collision to the negative of their relative velocity before the collision. By this definition, the elasticity should always be positive:

$$k = \frac{\vec{v}_{2f} - \vec{v}_{1f}}{\vec{v}_{1i} - \vec{v}_{2i}}$$ (Equation 7.4: **Elasticity**)

The four categories of collisions can also be defined in terms of the elasticity.

Type of Collision	Kinetic Energy	Elasticity	Example
Super-elastic	$K_f > K_i$	$k > 1$	Carts are initially stationary, then pushed apart by a spring-loaded piston, as in Exploration 6.4. An explosion.
Elastic	$K_f = K_i$	$k = 1$	Carts with repelling magnets.
Inelastic	$K_f < K_i$	$k < 1$	Describes most collisions, such as two cars that make contact when colliding but that don't stick together.
Completely inelastic	$K_f < K_i$, and the objects stick together	$k = 0$	Carts with Velcro, as in Exploration 7-3, or chewing gum hitting the sidewalk.

Table 7.1: Collisions can be classified in terms of what happens to the kinetic energy or in terms of the elasticity. Note that, in an elastic collision, the fact that $k = 1$ can be obtained by combining the momentum conservation equation with the conservation of kinetic energy equation.

EXAMPLE 7.5 – An assist from gravity

Sending a space probe from Earth to another planet requires a great deal of energy. In many cases, a significant fraction of the probe's kinetic energy can be provided by a third planet, through a process known as a ***gravitational assist***. For instance, the Cassini-Huygens space probe launched on October 15, 1997, used four gravitational assists, two from Venus, one from Earth, and one from Jupiter, to speed it on its more than 1 billion km trip to Saturn, arriving there on July 1, 2004. We can treat a gravitational assist as an elastic collision, because the long-range interaction of the probe and the planet provides no mechanism for a loss of mechanical energy.

Figure 7.10: The Cassini-Huygens space probe while it was being assembled. The desk and chair at the lower left give a sense of the scale. Photo courtesy NASA/JPL-Caltech.

A space probe with a speed v is approaching Venus, which is traveling at a velocity V in the opposite direction. The probe's trajectory around the planet reverses the direction of the probe's velocity. (a) How fast does the probe travel away from Venus? (b) If $v = 1 \times 10^5$ m/s, and $V = 3.5 \times 10^5$ m/s, what is the ratio of the probe's final kinetic energy to its initial kinetic energy?

SOLUTION

Let's begin with a diagram of the situation, shown in Figure 7.11. Although we will analyze this situation as a collision, the objects do not make contact with one another.

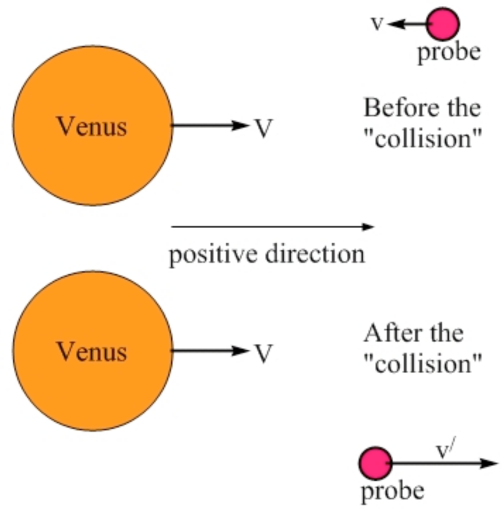

Figure 7.11: Before and After situations for the interaction between Venus and the space probe.

(a) The probe's speed depends on how far away it is from Venus. Because no distances were given, let's assume the probe has speed v when it is so far from Venus that the gravitational pull of Venus is negligible. We will work out the final velocity under the same assumption. This is an elastic collision. We could apply conservation of momentum and conservation of energy, but we were not given any masses and the resulting equations can be challenging to combine to find the final velocity of the probe. Let's try working with the elasticity k instead.

Because this collision is elastic, $k = 1$. The elasticity is the ratio of the final relative speed to the initial relative speed, so those two relative speeds must be equal for k to equal 1. Also, we can reasonably assume that the probe's mass is much smaller than the planet's mass and that the planet's motion is unaffected by its interaction with the probe. Thus, the planet's velocity is V in the direction shown both before and after the collision.

Defining the direction of the planet's velocity as the positive direction, plugging everything into the elasticity equation gives:

$$1 = \frac{\vec{v}_{2f} - \vec{v}_{1f}}{\vec{v}_{1i} - \vec{v}_{2i}} = \frac{V - v_{1f}}{-v - V} \quad ,$$

where both the numerator and denominator are negative.

Solving for the final speed of the probe gives $v_{1f} = 2V + v$.

(b) Substituting in the numbers gives a final speed of 8×10^5 m/s, an increase by a factor of 8 in speed. Kinetic energy is proportional to the square of the speed, so the probe's kinetic energy is increased by a factor of 64. A more formal way to show this relation is the following:

$$\frac{K_f}{K_i} = \frac{\frac{1}{2} m v_f^2}{\frac{1}{2} m v_i^2} = \frac{v_f^2}{v_i^2} = \frac{\left(8 x 10^5 \text{ m/s}\right)^2}{\left(1 x 10^5 \text{ m/s}\right)^2} = 64 \, .$$

The probe gains an enormous amount of energy, and it does so without requiring massive amounts of fuel to be burned. This is why probes to the outer planets are often first sent toward Venus, because the large increase in speed more than makes up for the extra distance traveled.

Related End-of-Chapter Exercises: 11, 54, 55.

Essential Question 7.5: In our analysis in example 7.5, we assumed that the planet's speed is constant. Is this absolutely correct? Is it a reasonable assumption?