Answer to Essential Question 7.3: All of them can be negative except for the kinetic energy, which can't be negative because $K = (1/2)mv^2$ and neither the square of the speed nor the mass can be negative. What is key is how the energies change, not what the values of the energies are.

7-4 Momentum and Collisions

Let's extend our understanding of momentum by analyzing a **collision**, which is an event in which two objects interact. As we learned in Chapter 6, Newton's third law tells us that, when no net external force acts on a system, the total momentum of the system is conserved. The momenta of the individual objects can change, but the total momentum of the system does not.

Generally, when we analyze a collision, we look at the situation immediately before the collision and compare it to the situation immediately after the collision. What happens during the collision itself can be interesting, and complicated. Fortunately, by using momentum we don't have to worry about such complications. The usual starting point in analyzing a collision is to write down a conservation of momentum equation reflecting the following relation:

Total momentum before the collision = total momentum after the collision.

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$ (Equation 7.2: **Momentum conservation**)

where the subscripts *i* and *f* stand for initial and final, and the two colliding objects are denoted by 1 and 2.

EXPLORATION 7.4 – Two carts collide…again

Two identical carts experience a collision on a horizontal track. Before the collision, cart 1 is moving at speed *v* to the right, directly toward cart 2, which is at rest. Immediately after the collision, cart 2 is moving with a velocity of $v/2$ to the right.

Step 1 - *What is the velocity of cart 1 immediately after the collision?* Let's begin, as usual, by drawing a diagram of the situation in Figure 7.9, showing the carts before and after the collision.

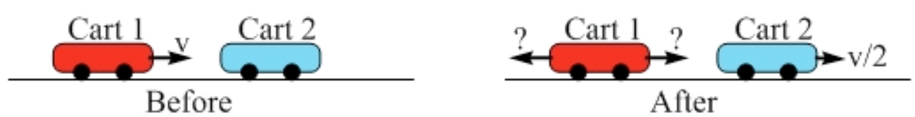

Figure 7.9: Two carts, immediately before and immediately after their collision.

The first step in applying equation 7.2 is to remember that momentum is a vector. Let's define right as the positive *x*-direction. We can say that each cart has a mass *m*, and we are given that $\vec{v}_{1i} = +v$, $\vec{v}_{2i} = 0$, and $\vec{v}_{2f} = +v/2$. Substituting all these terms into the conservation of momentum equation gives:

$$+mv = m\vec{v}_{1f} + m\frac{v}{2}$$

Dividing out a factor of *m* and solving for the velocity of cart 1 after the collision gives:

$$\vec{v}_{1f} = +\frac{v}{2}.$$

The two carts have the same velocity, and thus move together, after the collision. We could arrange this special case by attaching Velcro to both carts so they stick together. When the objects move together afterwards, we say that the collision is ***completely inelastic***.

Step 2 - ***Is kinetic energy conserved in this collision?*** Kinetic energy does not have to be conserved in a collision, although in certain special cases it is. Let's see what happens to the kinetic energy in this case.

Before the collision: $K_i = \frac{1}{2}mv_{1i}^2 + \frac{1}{2}mv_{2i}^2 = \frac{1}{2}mv^2$.

After the collision: $K_f = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2 = \frac{K_i}{2}$.

In this case, only 50% of the kinetic energy from before the collision is in the system as kinetic energy after the collision. The total energy has to be conserved, but in this case, some of the kinetic energy of cart 1 before the collision is transformed to other forms of energy (such as thermal energy, which is energy associated with the motion of molecules, and sound energy) in the collision process.

Step 3 - ***What is the velocity of the system's center of mass before the collision?*** By dividing both sides of Equation 6.4, for the position of the center of mass, by a time interval, Δt, and using the definition of velocity, we can obtain an equation for the velocity of the center of mass:

$$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$ (Equation 7.3: **Velocity of the center of mass**)

The m's represent the masses of the various pieces of the object or system. The terms in the numerator on the right represent the momenta of the individual parts of the system, so the equation really says that the total momentum of the system is the vector sum of the momenta of its parts, which seems sensible.

Applying the equation to the two-cart system before the collision gives:

$$\vec{v}_{CM,i} = \frac{+mv + m\times 0}{m+m} = +\frac{v}{2}.$$

This result makes sense because the center of mass is halfway between the carts, so the center of mass covers half the distance as cart 1 does in the same time.

Step 4 - ***What is the velocity of the system's center of mass after the collision?*** Applying equation 7.3 to the system after the collision gives:

$$\vec{v}_{CM,f} = \frac{+m\frac{v}{2} + m\frac{v}{2}}{m+m} = +\frac{v}{2}.$$

It should come as no surprise that the velocity of the center of mass after the collision is the same as the velocity of the center of mass before the collision. Rather, this result is expected as a consequence of momentum conservation. In short, the center of mass does not even register that a collision has taken place.

Key ideas: In a collision, in general, the system's momentum is conserved while the system's kinetic energy is not necessarily conserved. In addition, in general, the motion of the system's center of mass is unaffected by the collision. **Related End-of-Chapter Exercises: 25 – 28.**

Essential Question 7.4: Under what condition is the momentum of a system conserved in a collision?