Answer to Essential Question 7.2: If there is no friction, the energy analysis is path independent, so we treat all paths the same. If friction does act on the object, however, even energy is hard to apply, because then the work done by friction depends on the path.

## 7-3 Energy Bar Graphs: Visualizing Energy Transfer

Energy conservation is a powerful tool. To make it easier for us to use this tool, it can be useful to use a visual aid to keep track of the various types of energy. One way to visualize how energy in a system is transformed from one type of energy to another is to use energy bar graphs. Consider the following example.

EXAMPLE 7.3A – Learning to use energy bar graphs
Consider a ball that you throw straight up into the air, and neglect air resistance. Define the gravitational potential energy to be zero at the point from which you release it. Draw energy bar graphs to show how the ball's mechanical energy is divided between kinetic energy and gravitational potential energy at the following points: (a) the point you release it; (b) the point halfway between the release point and the maximum-height point, on the way up; (c) the maximum-height point; (d) the point halfway between the release point and the maximum height, on the way down; and (e) the release point, on the way down.

SOLUTION
The five sets of bar graphs are shown in Figure 7.5. The vertical position of the ball is indicated above each set of graphs, making it clearer why the second and fourth sets of graphs are the same, and why the first and last sets are the same. As the ball rises, its kinetic energy is transformed into potential energy, reaching 100% potential at the maximum height point. As the ball falls, the potential energy transforms back to kinetic energy.
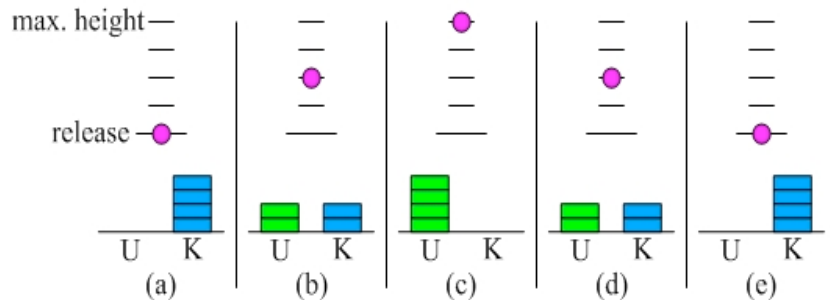

Figure 7.5: Energy bar graphs for a ball going straight up and down. The ball's corresponding vertical position is shown above each of the bar

Related End-of-Chapter Exercises: 21, 22.

In this case, the total mechanical energy is conserved. Let's do another example to see how bar graphs are used to depict a situation in which energy is not conserved.

EXAMPLE 7.3B – Extending the use of energy bar graphs
A string is tied to a block that has a mass of 2.0 kg. As shown in Figure 7.6, the string passes over a pulley, and you hang onto the end of the string to prevent the block from moving. Initially, the block is 1.0 m above the ground. You then pull down on the string so the block accelerates upward at a constant rate of 4.0 m/s². Use $g = 10$ m/s², and define the zero for gravitational potential energy to be at ground level. Draw bar graphs to show the block's gravitational potential energy, kinetic energy, total mechanical energy, and the work you have done on the block (a) at the instant the block starts to move, and (b) 0.50 s after the block starts to move.
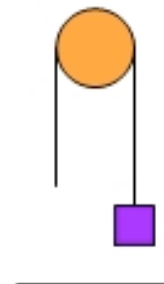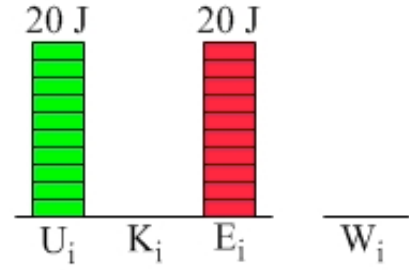

Figure 7.6: The system of the string, pulley, and block. You hold the string.

Figure 7.7: Energy bar graphs showing the energy at the start of the motion. Each small rectangle represents an energy of 2 J. E is the total mechanical energy.

SOLUTION

(a) Let's define up to be the positive y-direction. Initially, the block is at rest and you have not done any work, so the energy is all gravitational potential energy. This relation is written

$U_i = mgy_i = (2.0 \text{ kg}) \times (10 \text{ m/s}^2) \times (1.0 \text{ m}) = 20 \text{ J}$. Because there is no kinetic energy, the block's total mechanical energy is 20 J. The bar graphs for the initial energies are shown in Figure 7.7.

(b) To find the block's gravitational potential energy at t = 0.50 s, we must determine how far off the ground the block is at that time. Because the acceleration is constant, we can use a constant-acceleration equation:

$$\vec{y}_{t=0.5} = \vec{y}_i + \vec{v}_{iy}t + \frac{1}{2}\vec{a}_y t^2 = +1.0 \text{ m} + 0 + \frac{1}{2}\left(+4.0 \text{ m/s}^2\right) \times (0.50 \text{ s})^2 = +1.5 \text{ m} .$$

This y-position corresponds to a gravitational potential energy of:

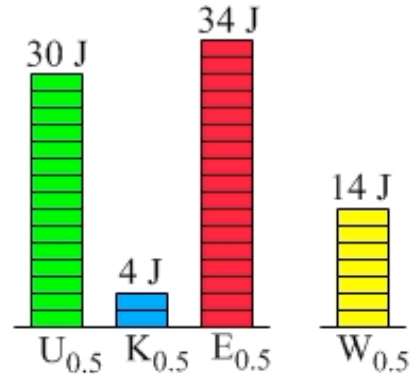$$U_{t=0.5} = mgy_{t=0.5} = (2.0 \text{ kg}) \times (10 \text{ m/s}^2) \times (+1.5 \text{ m}) = +30 \text{ J} .$$

To find the kinetic energy at t = 0.50 s, let's find the speed of the block. We can use a constant-acceleration equation to find the block's velocity at t = 0.50 s and then take the magnitude to get the speed.

$$\vec{v}_{y,t=0.5} = \vec{v}_{iy} + \vec{a}_y t = 0 + \left(+4.0 \text{ m/s}^2\right) \times (0.50 \text{ s}) = +2.0 \text{ m/s} .$$

The kinetic energy is then

$$K_{t=0.5} = \frac{1}{2}mv^2 = \frac{1}{2}(2.0 \text{ kg}) \times (2.0 \text{ m/s})^2 = 4.0 \text{ J} .$$

Figure 7.8: Energy bar graphs showing the energy of the block, and the work (W) you have done on the block up to that point, at t = 0.50 s.

The total mechanical energy is the sum of the potential energy and the kinetic energy, for a total of 34 J. This is 14 J larger than the initial mechanical energy, meaning that you must have done 14 J of work on the block. All the energies are represented by the bar graphs in Figure 7.8.

To summarize this section, the visualization technique of drawing energy bar graphs can be applied to systems in which mechanical energy is conserved, or even to systems when it is not conserved. Drawing these bar graphs is a good way to keep track of the different types of energy in a system.

Related End-of-Chapter Exercises: 23, 24.

Essential Question 7.3: In Example 7.3B, the bar graphs representing the various energies and the work are all either positive or zero. Could any of these be negative in some circumstances?