Answer to Essential Question 7.1: In the case of the book sliding on the table, we can apply either an energy analysis or a force analysis. Let's now compare these different methods.

7-2 Comparing the Energy and Force Approaches

In Example 7.1, there is no real advantage in using an energy analysis over a force analysis. In some cases, however, the energy approach is much easier than the force approach.

EXPLORATION 7.2A – Which ball has the higher speed?

Three identical balls are launched with equal speeds *v* from a height *h* above level ground. Ball A is launched horizontally, while the initial velocity of ball B is at 30º above the horizontal, and the initial velocity of ball C is at 45º below the horizontal. Rank the three balls, based on their speeds when they reach the ground, from largest to smallest. Neglect air resistance.

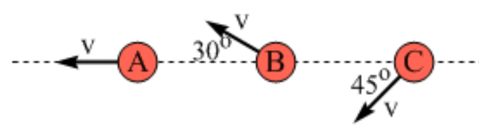

Figure 7.2: A diagram showing the directions of the initial velocities of the three balls in Exploration 7.2A.

Step 1 – ***Sketch a diagram of the situation.*** See Figure 7.2.

Step 2 – ***Briefly describe how to solve this problem using methods applied in earlier chapters.*** Consider the projectile-motion analysis we applied in chapter 4. For each ball, we would break the initial velocity into components, determine the *y*-component of the final velocity using one of the constant-acceleration equations, and then find the magnitude of the final velocity by using the Pythagorean theorem. We would have to go through the process three times, once for each ball.

Step 3 – ***Instead, solve the problem using an energy approach.*** Our starting point for energy is always the conservation of energy equation, $K_i + U_i + W_{nc} = K_f + U_f$. There is no air resistance, so $W_{nc} = 0$. If we define the zero for gravitational potential energy as the ground level, then $U_f = 0$, and $U_i = mgh$, where *m* is the mass of a ball (each ball has the same mass). Substituting this expression into the conservation of energy equation gives: $K_i + mgh = K_f$.

Both terms on the left are the same for all three balls. The balls have the same initial kinetic energy and they experience the same change in potential energy. Thus, all three balls have identical final kinetic energies. Because $K_f = (1/2)mv_f^2$, and the balls have equal masses, the final speeds are equal. Based on one energy analysis that works for all three balls, instead of three separate projectile-motion analyses, the ranking of the balls based on final speed is A=B=C.

Step 4 – ***Would the answer be different if the balls had unequal masses?*** Starting from $K_i + mgh = K_f$, and using the definition of kinetic energy, we can show that mass is irrelevant:

$$\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv_f^2 .$$

Factors of mass cancel, giving: $v_f = \sqrt{2gh + v_i^2}$.

So, the balls have the same final speed even if their masses are different.

Key idea for solving problems: We now have two powerful ways of analyzing physical situations. We can either apply force ideas, or apply energy ideas. In certain situations the energy approach is simpler than the force approach. **Related End-of-Chapter Exercises: 6, 38.**

EXPLORATION 7.2B – Which cart wins the race?

As shown in Figure 7.3, two identical carts have a race on separate tracks. Cart A's track follows a straight path sloping down, while cart B's track dips down below A's just after the start and rises up to meet A's again just before the finish line. If the carts are released at the same time, which cart reaches the end of the track first? Make a prediction, and justify your answer.

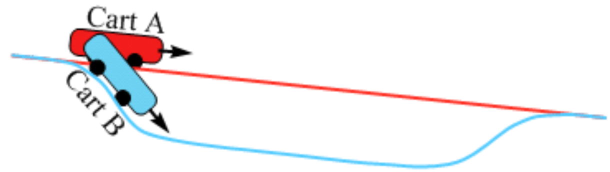

Figure 7.3: A race between two identical carts. Cart A's track has a constant slope, while cart B's track dips down below A's before rising up to meet A's again at the end.

After considering the three balls of Exploration 7.2A, it is tempting to predict that the race will end in a tie. An energy analysis, for instance, follows the same logic as that in Exploration 7.2A, except that the analysis in this case is simplified by the initial kinetic energy being zero. Once again, energy tells us that the carts should arrive at the finish line with the same speed. Having the same speed does not mean that they arrive at the same time, however.

Another popular answer is that A wins the race because B travels farther. In actuality, for most tracks, cart B wins the race. Cart A gradually picks up speed as it loses potential energy. In contrast, cart B immediately drops below A, transforming potential energy into kinetic energy, and reaching a speed larger than that of cart A. The carts then travel along parallel paths, with B always moving faster than A. Even while B slows as it climbs the hill near the end, it is traveling faster than A. The larger distance B travels is more than made up for by B's larger average speed.

Key idea about energy and time: Energy can be a powerful concept, but energy generally gives us no direct information about time. **Related End-of-Chapter Exercises: 4, 35.**

Let's compare and contrast the energy approach with the force approach. Energy can be a very effective tool, because in many cases we only have to consider the initial and final states and we don't have to worry about how the system gets from one state to the other. On the other hand, energy tells us nothing about the time it takes to get from one state to another. In Exploration 7.2B, for instance, the three balls reach the ground at different times, but we would have to use forces, and the constant-acceleration equations, to find those times. Energy is also a scalar, so it tells us nothing about direction. Energy is perfect, however, for connecting speed and position.

If we want to learn about time, or about the direction of a vector, analyzing forces is a better approach. So far, though, we are limited to applying force concepts to situations in which the net force is constant, when we can apply the constant-acceleration equations. We will go beyond this in Chapter 8 but, at the level of physics we are concerned with in this book, we will always be limited in how far we can go with forces. A good example of the limitations of force is shown in Figure 7.4, where an object slides from point A to point B along various paths. If the object comes down path 1, the straight line connecting A and B, we can use forces or energy to analyze the motion, even if friction acts as the object slides. If the object slides along a path other than path 1, then we can't get far with forces. The difficulty is that the force approach is ***path dependent*** – the forces applied to the object depend on the path, and the forces change if the object moves along a different path.

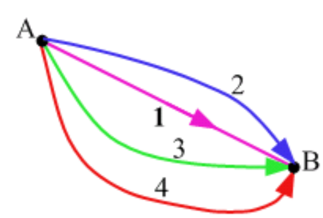

Figure 7.4: Various paths for an object to slide along in traveling from A to B.

Essential Question 7.2: Could we use energy to analyze the motion along paths 2, 3, or 4 in Figure 7.4?