

7-1 The Law of Conservation of Energy

In Chapter 6, we looked at the work-energy relationship: $W_{net} = \Delta K$. If we break the net work up into two pieces, W_{con} , the work done by conservative forces (such as gravity), and W_{nc} , the work done by non-conservative forces (such as tension or friction), then we can write the equation as: $W_{con} + W_{nc} = \Delta K$. Recall that the work done by a conservative force does not depend on the path taken between points. The work done by non-conservative forces is path dependent, however.

As we did in Chapter 6 with the work done by gravity, the work done by any conservative force can be expressed in terms of potential energy, using $W_{con} = -\Delta U$. We can now write the work-energy equation as

$$-\Delta U + W_{nc} = \Delta K. \quad (\text{Equation A})$$

Let's use i to denote the initial state and f to denote the final state. The change in a quantity is its final value minus its initial value, so we can use $\Delta K = K_f - K_i$ and $\Delta U = U_f - U_i$. Substituting these expressions into Equation A gives $-(U_f - U_i) + W_{nc} = K_f - K_i$. With a bit of re-arranging to make everything positive, we get Equation 7.1, below.

Equation 7.1 expresses a basic statement of the **Law of Conservation of Energy**:
"Energy can neither be created nor destroyed, it can only be changed from one form to another."

$$K_i + U_i + W_{nc} = K_f + U_f \quad (\text{Equation 7.1: Conservation of energy})$$

The law of conservation of energy is so important that we will use it in Chapters 8, 9, and 10, as well as in many chapters after that. With equation 7.1, we have the only equation we need to solve virtually any energy problem. Let's discuss its five different components.

K_i and K_f are the initial and final values of the kinetic energy, respectively.

U_i and U_f are the initial and final values of the potential energy, respectively.

W_{nc} is the work done by non-conservative forces (such as by the force of friction).

The conservation of energy equation is very flexible. So far, we have discussed one form of kinetic energy, the translational kinetic energy given by $K = (1/2)mv^2$. When we get to Chapter 11, we will be able to build rotational kinetic energy into energy conservation without needing to modify equation 7.1. Similarly, no change in the equation will be necessary when we define a general form of gravitational potential energy in Chapter 8, and define spring potential energy in Chapter 12. It will only be necessary to expand our definitions of potential and kinetic energy.

Mechanical energy is the sum of the potential and the kinetic energies. If no net work is done by non-conservative forces (if $W_{nc} = 0$), then mechanical energy is conserved. This is the **principle of the conservation of mechanical energy**.

EXAMPLE 7.1 – A frictional best-seller

A popular book, with a mass of 1.2 kg, is pushed across a table. The book has an initial speed of 2.0 m/s, and it comes to rest after sliding through a distance of 0.80 m.

- What is the work done by friction in this situation?
- What is the average force of friction acting on the book as it slides?

SOLUTION

(a) As usual, we should draw a diagram of the situation and a free-body diagram. These diagrams are shown in Figure 7.1. Three forces act on the book as it slides. The normal force is directed up, at 90° to the displacement, so the normal force does no work. The effect of the force of gravity is accounted for via the potential energy terms in equation 7.1, but the gravitational potential energy does not change, because the book does not move up or down. The only force that affects the energy is the force of friction. The work done by friction is accounted for by the W_{nc} term in the conservation of energy equation.

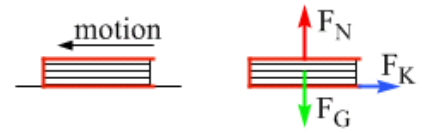


Figure 7.1: A diagram of the sliding book and a free-body diagram showing the forces acting on it as it slides.

So, the five-term conservation of energy equation, $K_i + U_i + W_{nc} = K_f + U_f$, can be reduced to two terms, because $U_i = U_f$ and the final kinetic energy $K_f = 0$. We are left with:

$$K_i + W_{nc} = 0 \quad \text{so, } W_{nc} = -K_i = -\frac{1}{2}mv_i^2 = -\frac{1}{2}(1.2 \text{ kg})(2.0 \text{ m/s})^2 = -2.4 \text{ J}.$$

The work done by the non-conservative force, which is kinetic friction in this case, is negative because the force of friction is opposite in direction to the displacement.

(b) To find the force of kinetic friction, F_K , use the definition of work. In this case, we get: $W_{nc} = F_K \Delta r \cos\theta = -F_K \Delta r$.

$$\text{Solving for the force of friction gives } F_K = -\frac{W_{nc}}{\Delta r} = -\frac{-2.4 \text{ J}}{0.80 \text{ m}} = 3.0 \text{ N}.$$

Related End-of-Chapter Exercises: 37, 40, 42.

A general method for solving a problem involving energy conservation

This general method can be applied to a wide variety of situations.

- Draw a diagram of the situation. Usually, we use energy to relate a system at one point, or instant in time, to the system at a different point, or a different instant.
- Write out equation 7.1, $K_i + U_i + W_{nc} = K_f + U_f$.
- Choose a level to be the zero for gravitational potential energy. Defining the zero level so that either U_i or U_f (or both) is zero is often best.
- Identify the terms in the equation that are zero.
- Take the remaining terms and solve.

Essential Question 7.1: Did we have to solve Example 7.1 using energy ideas, or could we have used forces and Newton's second law instead?