Answer to Essential Question 7.7: The balls don't stick together, so we know the collision is not completely inelastic. One way to classify the collision is to find the elasticity, *k* (see equation 7.4).

$$k = \frac{\vec{v}_{Bf} - \vec{v}_{Af}}{\vec{v}_{Ai} - \vec{v}_{Bi}} = \frac{1.0 \text{ m/s} - (-2.0 \text{ m/s})}{4.0 \text{ m/s} - 0} = 0.75 .$$

The fact that *k* is less than 1 means the collision is inelastic. We can confirm this result by looking at the kinetic energy before and after the collision.

$$K_i = \frac{1}{2} m_A v_{Ai}^2 + \frac{1}{2} m_B v_{Bi}^2 = \frac{1}{2} \times (4.0 \text{ kg}) \times (4.0 \text{ m/s})^2 + 0 = 32 \text{ J} .$$

$$K_f = \frac{1}{2} m_A v_{Af}^2 + \frac{1}{2} m_B v_{Bf}^2 = \frac{1}{2} \times (4.0 \text{ kg}) \times (2.0 \text{ m/s})^2 + \frac{1}{2} \times (24.0 \text{ kg}) \times (1.0 \text{ m/s})^2 ;$$

$$K_f = 8.0 \text{ J} + 12 \text{ J} = 20 \text{ J} .$$

The kinetic energy in the system after the collision is less than it is before the collision, so we have an inelastic collision.

Chapter Summary

Essential Idea about Conservation Laws

Many physical situations can be analyzed using forces, which we learned about in previous chapters, and/or by applying the fundamental concepts of conservation of momentum and conservation of energy, which we learned about in this chapter.

Comparing the Energy and Force Methods

The primary methods we use to analyze situations are to use forces and Newton's Laws, or to use energy conservation. Let's compare these two methods.

- The energy approach can be very effective, because we often just have to deal with the initial and final states and we don't have to account for the path taken by the system in going from one state to another, as we do with the force approach.
- The energy approach, by itself, does not give us any information about time, such as about how long it takes a system to move from one state to another. If you need to know about time, use a force analysis.
- Energy is a scalar. Thus energy, by itself, tells us nothing about direction. Force is a vector, and this it can give us information about direction.
- If W_{nc}, the work done by non-conservative forces, is zero, then the total ***mechanical energy*** (the sum of the kinetic and potential energies) is conserved.

A General Method for Solving a Problem Involving Energy Conservation

1. Draw a diagram of the situation. Usually, we use energy to relate a system at one point, or instant in time, to the system at a different point, or a different instant.
2. Apply energy conservation: $K_i + U_i + W_{nc} = K_f + U_f$. (Eq. 7.1)
3. Choose a level to be the zero for gravitational potential energy. Setting the zero level so that either U_i or U_f (or both) is zero is often best.
4. Identify the terms in the equation that are zero.
5. Take the remaining terms and solve.

Collisions and Momentum Conservation

In general, the momentum of a system is conserved in a collision, but the system's kinetic energy is often not conserved in a collision. In fact, one of the two ways in which we classify collisions is based on how the kinetic energy before the collision compares to that afterwards. The second way collisions can be classified is in terms of the ***elasticity***, k, which is the ratio of the relative speed of the colliding objects after the collision to their relative speed after the collision:

$$k = \frac{\vec{v}_{2f} - \vec{v}_{1f}}{\vec{v}_{1i} - \vec{v}_{2i}} \quad . \qquad \text{(Equation 7.4: \textbf{Elasticity})}$$

This equation is particularly useful when the collision is elastic and the relative velocity of the objects has the same magnitude before and after the collision.

The four collision categories are:

Type of Collision	Kinetic Energy	Elasticity
Super-elastic	$K_f > K_i$	$k > 1$
Elastic	$K_f = K_i$	$k = 1$
Inelastic	$K_f < K_i$	$k < 1$
Completely inelastic	$K_f < K_i$, and the objects stick together	$k = 0$

A General Method for Solving a Problem Involving a Collision

1. Draw a diagram of the situation, showing the velocity of the objects immediately before and immediately after the collision.
2. In a two-dimensional situation, set up a table showing the components of the momentum before and after the collision for each object.
3. Use momentum conservation: $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$. (Eq. 7.2)

   Apply equation 7.2 twice, once for each direction, in a two-dimensional situation). Account for the fact that momentum is a vector with + and – signs.
4. If you require an additional relationship (such as in the case of an elastic collision) use the elasticity relationship or write an energy-conservation equation.