Answer to Essential Question 6.6: No, gravitational potential energy really does not belong to an object. Rather, it is associated with the interaction between two objects, such as the interaction between the ball and the Earth in Exploration 6.6A. We will explore this idea further when we discuss gravity in more detail in Chapter 8.

6-7 Power

Let's say you are buying a new vehicle. While you are searching the Internet to compare the latest models, an advertisement for a fancy sports car catches your eye. You read that the car can go from rest to 100 km/h in under five seconds, considerably less time than it takes a base-model Honda Civic, for instance, to do the same. Then, when you tell your friend about what you're planning, he encourages you to buy a pickup truck. The truck and the Civic have similar accelerations, but the truck can achieve that acceleration while loaded down with bikes and kayaks. What is the difference between these vehicles? Their engines can all do work, but an important difference between them is the rate at which they do work.

The ability to do work quickly is something that we celebrate. For instance, in many Olympic events, the gold medal goes to the individual who can do more work, and/or do work in less time, than the other athletes. Once again, we should name this important concept.

Power is the rate at which work is done. The unit of power is the watt, and 1 W = 1 J/s.

$$P = \frac{Work}{\Delta t} = \frac{F \Delta r \cos\theta}{\Delta t} = F v \cos\theta \ , \qquad \text{(Equation 6.11: **Power**)}$$

where θ is the angle between the force and the velocity.

EXAMPLE 6.7 – Climbing the hill

A car with a weight of mg = 16000 N is climbing a hill that is 120 m long and rises 30 m vertically. The car is traveling at a constant velocity of 72 km/h. In addition to having to contend with the component of the force of gravity that acts down the slope, the car also has to deal with a constant 1000 N in resistive forces as it climbs.

(a) What is the power provided to the drive wheels by the car's engine?

(b) The power unit the horsepower was first used by James Watt in 1782 to compare steam engines and horses. What is the car's power in units of horsepower, where 1 hp = 746 W?

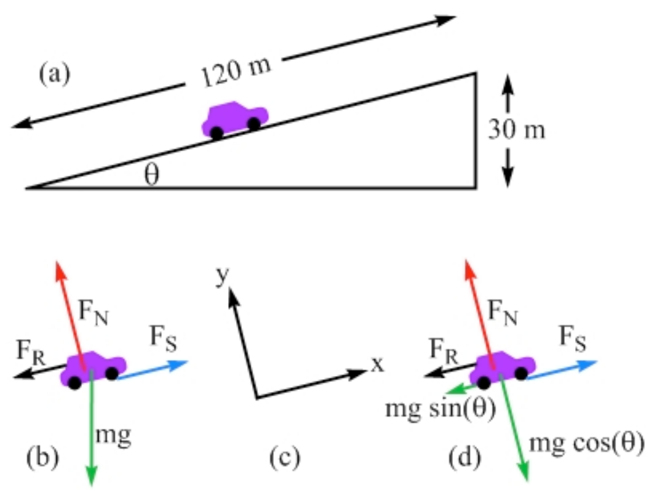

Figure 6.16: (a) A diagram for the car climbing the hill, along with (b) a free-body diagram, (c) an appropriate coordinate system, aligned with the slope, and (d) a revised free-body diagram, with all forces aligned with the coordinate system.

SOLUTION

Let's begin by sketching a diagram of the situation (see Figure 6.16), along with a free-body diagram. If we use a coordinate system aligned with the slope, with the positive x-direction up the slope, we can re-draw the free-body diagram with all the forces parallel to the coordinate axes. Doing so involves breaking the force of gravity into components.

(a) Let's assume that this case is typical and the tires do not slip on the road surface as the car climbs the hill. If so, the force propelling the car up the slope is a static force of friction, much like the force propelling you forward when you walk is a static force of friction. This force of static friction, directed up the hill, must balance the sum of the 1000 N resistive force and the component of the force of gravity acting down the hill, which is $mg\sin\theta$. The value of $\sin\theta$ can be found from the geometry of the hill:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{30\text{ m}}{120\text{ m}} = \frac{1}{4}.$$

The net force directed up the hill is:

$$F_{up} = 1000\text{ N} + mg\sin\theta = 1000\text{ N} + 16000 \times \frac{1}{4} = 1000\text{ N} + 4000\text{ N} = 5000\text{ N}.$$

The car's velocity is also directed up, so, if we multiply the force by the speed, we get the power. The speed has to be expressed in units of m/s, however. So, we perform the conversion:

$$72\,\frac{\text{km}}{\text{h}} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ h}}{3600\text{ s}} = 20\text{ m/s}.$$

The power associated with the drive wheels is:

$$P = F\,v\cos\theta = F\,v = 5000\text{ N} \times 20\text{ m/s} = 100000\text{ W}.$$

(b) Converting watts to horsepower gives:

$$100000\text{ W} \times \frac{1\text{ hp}}{746\text{ W}} = 134\text{ hp}.$$

Not every car is capable of putting out that much power, but many cars are, so that's a reasonable value. The car is probably working at close to its maximum power output, however.

Related End-of-Chapter Exercise: 57, 58.

Question: A typical adult takes in about 2500 nutritional Calories of food energy in a day. Using the fact that 1 Calorie is equivalent to 1000 calories, and that 1 calorie is equivalent to 4.186 J, show that a typical adult takes in about 1 x 10^7 J worth of food energy in a day.

Answer: Much like converting from watts to horsepower, this is an exercise in unit conversion.

$$2500\text{ Cal} \times \frac{1000\text{ cal}}{1\text{ Cal}} \times \frac{4.186\text{ J}}{1\text{ cal}} = 1.05 \times 10^7\text{ J}.$$

Essential Question 6.7: As we have just shown, a typical adult takes in about 1 x 10^7 J of food energy in a day. Assuming this energy equals the work done by the person in a day, what average power output does this correspond to? Compare this power to the power output of a world-class cyclist, who can sustain a power output of 500 W for several hours.